Math 5637
Risk Theory
Fall 2014
Final Examination
December 5 - 10, 2014

Due back to me by 5 PM on Wednesday, December 10, in my mailbox, under my door, or by email. You may consult with any written source, including textbooks, solution manuals, notes, websites, or anything else in writing. Remember Appendix A. Do NOT consult with any other person. Doing so will be grounds for failing the course. The five questions will be equally weighted in the grading.

1. Individual loss amounts (ground up) this year follow a two parameter Pareto distribution with $\alpha = 3$ and expected value 1000. Next year you confidently expect loss amounts to inflate by 5% uniformly across all losses. What will be the standard deviation next year for loss amounts that are limited to $2,000$ per loss with the limited loss amount further subjected to a 200 deductible per loss? Please answer for the "per loss" variable, not the "per payment" variable. (HINT: use the fact that $\beta(3, 1; u) = u^3$ to help your calculations.)

2. Write down formulas for the mean and first six central moments of the compound Poisson random variable

   $S = X_1 + ... + X_N$ with the X_j being i.i.d. copies of the random variable X

   in terms of the Poission frequency λ of N and the first six raw moments of X.

3. Rank the following distributions in order of heaviness of their tails: lognormal, loglogistic, and inverse gamma. Justify your rankings with precise demonstrations.

4. If N is a compound Poisson-Poisson random variable with parameters $\lambda = .25$ for the primary variable and $\lambda = 1$ for the secondary variable, and if X can be approximated by a negative binomial random variable with $\beta = 2$ and $r = .5$ then calculate numerical values for the first 5 probabilities $\mathbb{P}[S = 0]$, $\mathbb{P}[S = 1]$, ... ,$\mathbb{P}[S = 4]$ for the random variable

   $S = X_1 + ... + X_N$ with the X_j being i.i.d. copies of the random variable X

   Hint: Think it through before you start calculating this one, and do the calculation in the easiest order.

5. Calculate a numerical value for the Conditional Tail Expectation (or TVaR) at probability level .95 for the inverse gaussian distribution with mean 1 and variance 1.