Math 5637
Risk Theory
Fall 2013
Final Examination
December 6 - 9, 2013

This is a take-home examination due back to me by 5 PM on Monday, December 9, in my department mailbox, under my office door, or by email. You may consult with any written source, including textbooks, solution manuals, notes, websites, or anything else in writing. This includes Appendix A. Do NOT consult with any other person. Doing so would be grounds for a failing grade in the course. Please put your name on all pages submitted. Include all of your calculations and all of your reasoning, both to support your answer but also to have a chance for partial credit in the grading of an incorrect answer. The six questions will be equally weighted in the grading.

1. Calculate a numerical value for the Conditional Tail Expectation (or TVaR) at probability level .995 for the inverse gamma distribution with parameters $\alpha = 3$, $\theta = 5,000,000$.

2. You are consulting to the director of an emergency relief agency that is trying to manage its budget by putting limits on the amount of relief it provides in case of windstorm damages. Historical data on the damages caused by each windstorm that occurred between Nov. 1, 2009 and Oct. 31 2013 were well-fit by an inverse gamma distribution with $\alpha = 3$ and expected value $2,500,000$. You are confident that damages caused by windstorms are increasing at an annual rate of 10% per year, because of both inflating reconstruction/repair costs and increasing numbers of structures exposed to damage. What would you project to be the mean and the standard deviation for the damages up to, but not beyond, a limit of $7,500,000$ per storm caused by a windstorm that causes at least $500,000$ in damages and that occurs between Jan.1, 2014 and Dec. 31, 2014?

3. If N is a compound Poisson-Poisson random variable with parameters $\lambda = .25$ for the primary variable and $\lambda = 1$ for the secondary variable, and if X can be approximated by a negative binomial random variable with $\beta = 4$ and $r = .25$ then calculate numerical values for the first 5 probabilities $\mathbb{P}[S = 0]$, $\mathbb{P}[S = 1]$, ... ,$\mathbb{P}[S = 4]$ for the random variable

   $S = X_1 + ... + X_N$ with the X_j being i.i.d. copies of the random variable X

   Hint: Think it through before you start calculating this one, and do the calculation in the easiest order.

4. Write down a formula for the 7th central moment μ_{S7} of the compound Poisson-Poisson random variable $S = M_1 + ... + M_N$ in terms of the two Poisson parameters λ_N and λ_M.

5. A certain crime frequency variable is known to follow a Poisson distribution for any given neighborhood of the city, but the Poisson frequency λ varies among the neighborhoods. In fact, λ is distributed across the neighborhoods as the sum of 5 identically and independently distributed exponential variables, each with mean 0.04. Assume that 20% of all crimes are too minor to report to the FBI data base. (a) What are the mean, variance and third central moment of the frequency of crimes reported to the FBI data base? (b) What would be the mean, variance and third central moment of the frequency of crimes reported to the FBI data base if the original λ were fixed at a constant value equal to the mean of the distribution of the neighborhood λ's, instead of varying across the neighborhoods?

6. Rank the following distributions in order of heaviness of their tails: lognormal, loglogistic, and gamma. Justify your rankings with precise demonstrations.