Math 5637
Risk Theory
Fall 2012
Final Examination
December 7 -12, 2012

This is a take-home examination due back to me by 5 PM on Wednesday, December 12, in my department mail box, under my office door, or by email. You may consult with any written source, including textbooks, solution manuals, notes, websites, or anything else in writing. Don't forget to use the Appendices in the textbook. Having so consulted, write the answers yourself, don't just copy an answer from somewhere else.

Do NOT under any circumstances consult with any other person, which would result in a failing grade for the course. Do NOT under any circumstances electronically cut and paste an answer from a source or electronically copy a spreadsheet that you did not create yourself, which will result in a failing grade for the course.

Please put your name on all pages submitted. Include all of your calculations and all of your reasoning, both to support your answer and to have a chance for partial credit on incorrect answers. The six questions will be equally weighted in the grading.

1. Derive the coeficient of skewness for each of the following distributions: negative binomial, binomial, Poisson. (You may start by assuming that you know any probability generating functions that you need but do not assume that any other generating functions or moments are known without deriving them, possibly from probability generating functions or by any other means.)

2. The frequency of risk to landslides is known to follow a Poisson distribution for any given mountainside, but the Poisson frequency λ varies among the different mountainsides exposed to the risk. In fact, λ is distributed across the different mountainsides as the sum of 5 identically and independently distributed exponential variables, each with mean 0.04. Of all landslides, 20% of them do not block roadways and can be ignored by traffic engineers. (a) What are the mean, variance and third central moment of the frequency of landslides that do block roadways and cannot be ignored by traffic engineers? (b) What would be the mean, variance and third central moment of the frequency of landslides that do block roadways and cannot be ignored if the original λ were fixed at a value equal to the mean of the distribution of λ, instead of varying across the mountainsides?

3. Show that the following four compound distributions are of the same form (i.e. they represent the same form of distribution) but possibly with different parameters: geometric-geometric; Bernoulli-geometric; zero-truncated geometric; zero-modified geometric. (Bernoulli is binomial for a single trial)

4. If $S_e(u)$ is the decumulative distribution function for the equilibrium distribution of a random variable X, $e(u)$ is the mean excess loss function for X, and $CTE(u)$ is the condition tail expectation of X at u, i.e. the $TVaR$ of X at u, prove that $CTE(u) \leq \frac{e(u)}{S_e(u)}$.

5. Individual loss amounts (ground up) this year follow an inverse Weibull distribution with $\tau = 3$ and expected value 1000. Next year you confidently expect loss amounts to inflate by 10% uniformly across the board. What will be the standard deviation next year for payment amounts that are subject to a 200 deductible per loss and limited to a maximum payment of $2,000$ per loss? Please answer for the "per loss" variable, not the "per payment" variable.

6. Start with the log-logistic random variable described in Appendix A. You can assume that $\theta = 1$. Perform a $CTM(k)$ transformation on it. Identify the resulting random variable, including the values for its parameters in Appendix A.