Math 5637
Risk Theory
Fall 2011
Final Examination
December 9 -12, 2011

This is a take-home examination due back to me by 4 PM on Monday, December 12, in my department mail box, under my office door, or by email. You may consult with any written source, including textbooks, solution manuals, notes, websites, or anything else in writing. Don't forget to use the Appendices in the textbook. Having so consulted, write the answers yourself.

Do NOT under any circumstances consult with any another person, which would result in a failing grade for the course. Do NOT under any circumstances electronically cut and paste an answer from a source or electronically copy a spreadsheet that you did not create yourself, which will result in a failing grade for the course.

Please put your name on all pages submitted. Include all of your calculations and all of your reasoning, both to support your answer and to have a chance for partial credit on incorrect answers. The four questions will be equally weighted in the grading.

1. Rank the following distributions in order of heaviness of their tails: lognormal, loglogistic, and gamma. Show why your ranking is correct.

2. Individual loss amounts (ground up) this year follow an inverse Weibull distribution with $\tau = 3$ and expected value 1000. Next year you confidently expect loss amounts to inflate by 10% uniformly across the board. What will be the standard deviation next year for payment amounts that are subject to a 250 deductible per loss and limited to a maximum payment of $2,250$ per loss? Please answer for the "per payment" variable, not the "per loss" variable.

3. Start with a loglogistic random variable with $\gamma = 1$. Apply a conditional tail moment transformation $CTM(k)$ to it. Identify the resulting random variable, including either a cdf, ddf, or pdf to justify your identification.

4. Write down a formula for the 4th central moment μ_{S4} of the compound Negative Binomial random variable $S = X_1 + ... + X_N$ in terms of the mean μ_X and the central moments μ_{Xk} of X for $2 \le k \le 4$ and the parameters r and β of the Negative Binomial random variable N. .