Math 5637
Risk Theory
Fall 2010
Final Examination
December 10 -15, 2010

This is a take-home examination due back to me by 5 PM on Wednesday, December 15, in my department mail box, under my office door, or by email. You may consult with any written source, including textbooks, solution manuals, notes, websites, or anything else in writing. Don't forget to use the Appendices in the textbook. Do NOT under any circumstances consult with another person, which would result in a failing grade for the course. Please put your name on all pages submitted. Include all of your calculations and all of your reasoning, both to support your answer and to have a chance for partial credit on incorrect answers. The five questions will be equally weighted in the grading.

1. Let $S = X_1 + ... + X_N$ where the X's are identically and independently distributed exponential random variables with mean 10 and N is a Poisson random variable with variance 3. Use a discrete approximation for X that rounds to the nearest half-integer value (0.5, 1.5, 2.5, ...), i.e. assign the probability for other values of X to the nearest half-integer value of X. Calculate the corresponding approximation for (a) the net stop-loss premium $\mathbb{E}\left[(S-2)_+\right]$, (b) the coefficient of variation of the stop-loss variable $(S-2)_+$ and (c) the conditional tail expectation $CTE_{F_S(2)}(S)$ at the quantile $F_S(2)$. Make your answers accurate to at least two decimal places.

2. Answer questions 1, (a), (b), and (c) without using the rounding approximation for X. Instead, use a discrete approximation for S directly that rounds to the nearest half-integer value, assigning probability for other values of S to the nearest half-integer value of S. To get the required probabilities for S take enough terms of a series summation for the S probabilities to get at least two decimal accuracy. (EXTRA CREDIT OPPORTUNITY: You don't have to do it this way, but I'll give you extra credit if you do this problem using only the series approximation when you need probabilities for S and using no rounding to half-integer values at all. Instead use a linear approximation to the surface interpretation areas and weighted areas ... volumes ... that you need.)

3. A certain event frequency variable follows a Poisson distribution for each member of a population at risk to the event, but the Poisson frequency λ varies across the population at risk. λ is distributed as the sum of 10 identically and independently distributed exponential random variables, each with mean 0.02. However, 20% of all events are almost unnoticeable and are never reported. (a) What are the mean, variance, and coefficient of skewness of the random variable representing the frequency of reported

events? (b) What would the mean, variance, and coefficient of skewness of the random variable representing the frequency of reported events be if the original λ were fixed at a value equal to the mean of the original random λ, rather than varying randomly across the population exposed to the risk?

4. Write down a formula for the 7th central moment μ_{S7} of the compound Poisson random variable $S = X_1 + ... + X_N$ in terms of the means μ_N and μ_X and central moments μ_{Nk} and μ_{Xk} of N and X for $2 \leq k \leq 7$, where N is Poisson with parameter λ.

5. Individual loss amounts (ground up) this year follow a two parameter Pareto distribution with $\alpha = 3$ and expected value 1000. Next year you confidently expect loss amounts to inflate by 5% uniformly across all losses. What will be the standard deviation next year for loss amounts that are limited to $2,000$ per loss with the limited loss amount further subjected to a 200 deductible per loss? Please answer for the "per loss" variable, not the "per payment" variable. (HINT: when you get to the point of needing to evaluate $\beta(3, 1; u)$, don't despair. $\beta(3, 1; u) = u^3$ can be derived quickly, but that's not what I want to test you on, so just take that fact and use it without bothering to derive it.)