

Math 5637
Risk Theory
Fall 2009
Final Examination
December 11-16, 2009

This is a take-home examination due back to me by 5 PM on Wednesday, December 16, in my department mailbox, under my office door, or by email. You may consult with any written source, including textbooks, solution manuals, notes, websites, or anything else in writing. This includes Appendix A. Do NOT consult with any other person. Doing so would be grounds for a failing grade in the course. Please put your name on all pages submitted. Include all of your calculations and all of your reasoning, both to support your answer but also to have a chance for partial credit in the grading of an incorrect answer. The five questions will be equally weighted in the grading.

1. Write down formulas for the mean and first six central moments of the compound Poisson random variable

$S = X_1 + \dots + X_N$ with the X_j being i.i.d. copies of the random variable X

in terms of the Poisson frequency λ of N and the first six raw moments of X .

2. Calculate a numerical value for the Conditional Tail Expectation (or TVaR) at probability level .995 for the inverse gamma distribution with parameters $\alpha = 3$, $\theta = 5,000,000$.
3. You are consulting to the director of a federal government emergency relief agency that is trying to manage its budget by putting limits on the relief it provides to the states for storm damages. Historical data on the damages caused by each windstorm that occurred between Nov. 1, 2005 and Oct. 31 2009 were well-fit by an inverse gamma distribution with $\alpha = 3$ and expected value 2,500,000. You are confident that damages caused by a windstorm are increasing at an annual rate of 10% per year, because of both inflating reconstruction/repair costs and increasing numbers of structures exposed to damage. What would you project to be the mean and the standard deviation for the damages up to, but not beyond, 7,500,000 caused by a windstorm that causes at least 500,000 in damages and that occurs between Jan.1, 2010 and Dec. 31, 2010?

4. Write down formulas for the mean and first four central moments of the compound random variable

$S = X_1 + \dots + X_N$ with the X_j being i.i.d. copies of the random variable X

in terms of the mean and first four central moments of N and X .

5. If N is a compound Poisson-Poisson random variable with parameters $\lambda = .25$ for the primary variable and $\lambda = 1$ for the secondary variable, and if X can be approximated by a negative binomial random variable with $\beta = 4$ and $r = .25$ then calculate numerical values for the first 5 probabilities $\mathbb{P}[S = 0]$, $\mathbb{P}[S = 1]$, \dots , $\mathbb{P}[S = 4]$ for the random variable

$S = X_1 + \dots + X_N$ with the X_j being i.i.d. copies of the random variable X

Hint: Think it through before you start calculating this one, and do the calculation in the easiest order.