Math 5637
Risk Theory
Fall 2009
Final Examination Solutions
December 11-16, 2009

This is a take-home examination due back to me by 5 PM on Wednesday, December 16, in my department mailbox, under my office door, or by email. You may consult with any written source, including textbooks, solution manuals, notes, websites, or anything else in writing. This includes Appendix A. Do NOT consult with any other person. Doing so would be grounds for a failing grade in the course. Please put your name on all pages submitted. Include all of your calculations and all of your reasoning, both to support your answer but also to have a chance for partial credit in the grading of an incorrect answer. The five questions will be equally weighted in the grading.

1. Write down formulas for the mean and first six central moments of the compound Poisson random variable

   $S = X_1 + ... + X_N$ with the X_j being i.i.d. copies of the random variable X

   in terms of the Poisson frequency λ of N and the first six raw moments of X.

   SOLUTION: $C_S(z) = C_N(C_X(z)) = \ln M_N(\ln M_X(z)) = \ln P_N(e^{\ln M_x(z)}) = \ln P_N(M_X(z)) = \ln e^{\lambda(M_X(z)-1)} = \lambda(M_X(z)-1)$.

   So, $\kappa_{Sj} = \lambda\mu'_{Xj}$ for all j. In particular, for $j=1$, $\mu_S = \kappa_{S1} = \lambda\mu_X$. Now, in order to get μ_{Sj} in terms of κ_{Sj} for $j \geq 2$ write

$$\begin{aligned} \ln M_S(z) &= C_S(z) \\ M_S(z) &= e^{C_S(z)} \\ e^{-\mu_S z} M_S(z) &= e^{C_S(z)-\mu_S z} \\ \mathbb{E}\left[e^{(S-\mu_S)z}\right] &= e^{C_S(z)-\mu_S z} \\ \mu_{Sj} &= \frac{d^j}{dz^j} e^{C_S(z)-\mu_S z} \end{aligned}$$

   Now apply Faá's formula, noticing that the 1^{st} derivative of the exponent is 0 when $z=0$, so the entire $k=1$ column of the Faá displays can be taken to be 0. The Faá displays therefore reduce to:

$$j=2: \left|\begin{array}{ccc} & \mathbf{1} & \mathbf{2} \\ \mathbf{1} & 0 & 1 \end{array}\right| \quad j=3: \left|\begin{array}{cccc} & \mathbf{1} & \mathbf{2} & \mathbf{3} \\ \mathbf{1} & 0 & 0 & 1 \end{array}\right| \quad j=4: \left|\begin{array}{ccccc} & \mathbf{1} & \mathbf{2} & \mathbf{3} & \mathbf{4} \\ \mathbf{2} & 0 & 2 & 0 & 0 \\ \mathbf{1} & 0 & 0 & 0 & 1 \end{array}\right|$$

$$j=5:\left|\begin{array}{c|ccccc} & \mathbf{1} & \mathbf{2} & \mathbf{3} & \mathbf{4} & \mathbf{5} \\ \mathbf{2} & 0 & 1 & 1 & 0 & 0 \\ \mathbf{1} & 0 & & & & 1 \end{array}\right| \quad j=6:\left|\begin{array}{c|cccccc} & \mathbf{1} & \mathbf{2} & \mathbf{3} & \mathbf{4} & \mathbf{5} & \mathbf{6} \\ \mathbf{3} & 0 & 3 & 0 & 0 & 0 & 0 \\ \mathbf{2} & 0 & 1 & 0 & 1 & 0 & 0 \\ \mathbf{2} & 0 & 0 & 2 & 0 & 0 & 0 \\ \mathbf{1} & 0 & 0 & 0 & 0 & 0 & 1 \end{array}\right| \text{ giving us}$$

$$\begin{aligned}
\mu_{S2} &= \kappa_{S2} = \lambda\mu'_{X2} \\
\mu_{S3} &= \kappa_{S3} = \lambda\mu'_{X3} \\
\mu_{S4} &= 3\kappa_{S2}^2 + \kappa_{S4} = 3\lambda^2\mu'^2_{X2} + \lambda\mu'_{X4} \\
\mu_{S5} &= 10\kappa_{S2}\kappa_{S3} + \kappa_{S5} = 10\lambda^2\mu'_{X2}\mu'_{X3} + \lambda\mu'_{X5} \\
\mu_{S6} &= 15\kappa_{S2}^3 + 15\kappa_{S2}\kappa_{S4} + 10\kappa_{S3}^2 + \kappa_{S6} \\
&= 15\lambda^3\mu'^3_{X2} + 15\lambda^2\mu'_{X2}\mu'_{X4} + 10\lambda^2\mu'^2_{X3} + \lambda\mu'_{X6}
\end{aligned}$$

2. Calculate a numerical value for the Conditional Tail Expectation (or TVaR) at probability level .995 for the inverse gamma distribution with parameters $\alpha = 3$, $\theta = 5,000,000$.

SOLUTION: Referring to App. A when needed, including Theorem A.1:

$$\begin{aligned}
CTE(.995) &= \pi_{.995} + \mathbb{E}\left[(X-\pi_{.995})|X>\pi_{.995}\right] \\
&= \pi_{.995} + \frac{1}{S(\pi_{.995})}\mathbb{E}\left[(X-\pi_{.995})_+\right] \\
&= \pi_{.995} + \frac{1}{1-.995}\left\{\mathbb{E}\left[X\right] - \mathbb{E}\left[X\wedge\pi_{.995}\right]\right\} \\
&= \pi_{.995} + \frac{1}{1-.995}\left\{\frac{\theta\Gamma(2)}{\Gamma(3)} - \frac{\theta\Gamma(2)}{\Gamma(3)}\left[1-\Gamma(2;\frac{\theta}{\pi_{.995}})\right] - \pi_{.995}\Gamma(3;\frac{\theta}{\pi_{.995}})\right\} \\
&= \pi_{.995} + \frac{1}{1-.995}\left\{\frac{\theta}{2}\left(1-e^{-\frac{\theta}{\pi_{.995}}}\sum_{j=0}^{1}\frac{\left(\frac{\theta}{\pi_{.995}}\right)^j}{j!}\right) - \pi_{.995}\left(1-e^{-\frac{\theta}{\pi_{.995}}}\sum_{j=0}^{2}\frac{\left(\frac{\theta}{\pi_{.995}}\right)^j}{j!}\right)\right\}
\end{aligned}$$

where $\pi_{.995}$ is the solution of $.995 = 1-\Gamma(3;\frac{\theta}{\pi_{.995}}) = 1-\left(1-e^{-\frac{\theta}{\pi_{.995}}}\sum_{j=0}^{2}\frac{\left(\frac{\theta}{\pi_{.995}}\right)^j}{j!}\right) = e^{-\frac{\theta}{\pi_{.995}}}\sum_{j=0}^{2}\frac{\left(\frac{\theta}{\pi_{.995}}\right)^j}{j!}$. By trial and error, or EXCEL Solver, the solution for $\pi_{.995}$ is $\pi_{.995} = 14,798,881$ so

$$\begin{aligned}
CTE(.995) &= \pi_{.995} + \frac{1}{1-.995}\left\{\frac{\theta}{2}\left(1-e^{-\frac{\theta}{\pi_{.995}}}\sum_{j=0}^{1}\frac{\left(\frac{\theta}{\pi_{.995}}\right)^j}{j!}\right) - \pi_{.995}\left(1-.995\right)\right\} \\
&= \frac{1}{.005}\left\{\frac{\theta}{2}\left(1-e^{-\frac{\theta}{\pi_{.995}}}\sum_{j=0}^{1}\frac{\left(\frac{\theta}{\pi_{.995}}\right)^j}{j!}\right)\right\} = 22,855,889
\end{aligned}$$

3. You are consulting to the director of a federal government emergency relief agency that is trying to manage its budget by putting limits on the relief it provides to the states for storm damages. Historical data on the damages caused by each windstorm that occurred between Nov. 1, 2005 and Oct. 31 2009 were well-fit by an inverse gamma distribution with $\alpha = 3$ and expected value $2,500,000$. You are confident that damages caused by a windstorm are increasing at an annual rate of 10% per year, because of both inflating reconstruction/repair costs and increasing numbers of structures exposed to damage. What would you project to be the mean and the standard deviation for the damages up to, but not beyond, $7,500,000$ caused by a windstorm that causes at least $500,000$ in damages and that occurs between Jan.1, 2010 and Dec. 31, 2010?

SOLUTION: The average date of the observed windstorms was Nov. 1, 2007. The average date of the projected windstorms will be July 1, 2010, a period of 32 months exposure to the 10% annual inflation. (Anyone who used 30 months on the theory that the hurricane season usually ends on Oct. 31 was given full credit). For an inverse gamma $\mathbb{E}[X] = \frac{\theta\Gamma(\alpha-1)}{\Gamma(\alpha)}$ so $\mathbb{E}[X] = 2,500,000$ with $\alpha = 3$ implies that $\theta = 5,000,000$ and we are starting with the same distribution as in problem 2. Using App. A when needed:

$$\mathbb{E}\left[\left\{(1.1)^{\frac{32}{12}}X\right\} \wedge 7,500,000 \mid \left\{(1.1)^{\frac{32}{12}}X\right\} \geq 500,000\right] =$$

$$= \mathbb{E}\left[(1.1)^{\frac{32}{12}}\left\{X \wedge \left(\frac{7,500,000}{(1.1)^{\frac{32}{12}}}\right)\right\} \mid X \geq \frac{500,000}{(1.1)^{\frac{32}{12}}}\right]$$

$$= (1.1)^{\frac{32}{12}}\,\mathbb{E}\left[X \wedge \left(\frac{7,500,000}{(1.1)^{\frac{32}{12}}}\right) \mid X \geq \frac{500,000}{(1.1)^{\frac{32}{12}}}\right], \text{ so by surf. interp. with weight 1}$$

$$= (1.1)^{\frac{32}{12}}\left\{\frac{500,000}{(1.1)^{\frac{32}{12}}} + \frac{1}{\mathbb{P}\left[X \geq \frac{500,000}{(1.1)^{\frac{32}{12}}}\right]}\left(\mathbb{E}\left[X \wedge \left(\frac{7,500,000}{(1.1)^{\frac{32}{12}}}\right)\right] - \mathbb{E}\left[X \wedge \left(\frac{500,000}{(1.1)^{\frac{32}{12}}}\right)\right]\right)\right\}$$

$$= 500,000 + \frac{(1.1)^{\frac{32}{12}}}{\Gamma(3;\frac{(1.1)^{\frac{32}{12}}\theta}{500,000})}\left\{\frac{\theta\Gamma(2)}{\Gamma(3)}\left[1-\Gamma(2;\frac{(1.1)^{\frac{32}{12}}\theta}{7,500,000})\right] + \frac{7,500,000}{(1.1)^{\frac{32}{12}}}\Gamma(3;\frac{(1.1)^{\frac{32}{12}}\theta}{7,500,000})\right.$$

$$\left. - \frac{\theta\Gamma(2)}{\Gamma(3)}\left[1-\Gamma(2;\frac{(1.1)^{\frac{32}{12}}\theta}{500,000})\right] - \frac{500,000}{(1.1)^{\frac{32}{12}}}\Gamma(3;\frac{(1.1)^{\frac{32}{12}}\theta}{500,000})\right\} \text{ from App. A}$$

$$= 500,000 + \frac{(1.1)^{\frac{32}{12}}}{\Gamma(3;10\,(1.1)^{\frac{32}{12}})}\left\{2,500,000\left[1-\Gamma(2;\frac{2}{3}\,(1.1)^{\frac{32}{12}})\right] + \frac{7,500,000}{(1.1)^{\frac{32}{12}}}\Gamma(3;\frac{2}{3}\,(1.1)^{\frac{32}{12}})\right.$$

$$\left. - 2,500,000\left[1-\Gamma(2;10\,(1.1)^{\frac{32}{12}})\right] - \frac{500,000}{(1.1)^{\frac{32}{12}}}\Gamma(3;10\,(1.1)^{\frac{32}{12}})\right\}.$$

using Theorem A.1 gives

$$
\begin{aligned}
&= 500,000 + \frac{(1.1)^{\frac{32}{12}}}{1 - e^{-10(1.1)^{\frac{32}{12}}} \sum_{j=0}^{2} \frac{\left(10(1.1)^{\frac{32}{12}}\right)^j}{j!}} \Bigg\{ 2,500,000 e^{-\frac{2}{3}(1.1)^{\frac{32}{12}}} \sum_{j=0}^{1} \frac{\left(\frac{2}{3}(1.1)^{\frac{32}{12}}\right)^j}{j!} \\
&\quad + \frac{7,500,000}{(1.1)^{\frac{32}{12}}} \left[1 - e^{-\frac{2}{3}(1.1)^{\frac{32}{12}}} \sum_{j=0}^{2} \frac{\left(\frac{2}{3}(1.1)^{\frac{32}{12}}\right)^j}{j!} \right] \\
&\quad - 2,500,000 e^{-10(1.1)^{\frac{32}{12}}} \sum_{j=0}^{1} \frac{\left(10(1.1)^{\frac{32}{12}}\right)^j}{j!} - \frac{500,000}{(1.1)^{\frac{32}{12}}} \left[1 - e^{-10(1.1)^{\frac{32}{12}}} \sum_{j=0}^{2} \frac{\left(10(1.1)^{\frac{32}{12}}\right)^j}{j!} \right] \Bigg\} \\
&= 2,960,973.5 \text{ for the mean. (Alternatively, get } \Gamma\text{s from EXCEL or some other software.)}
\end{aligned}
$$

$$
\mathbb{E}\left[\left(\left\{ (1.1)^{\frac{32}{12}} X \right\} \wedge 7,500,000 \right)^2 \mid \left\{ (1.1)^{\frac{32}{12}} X \right\} \geq 500,000 \right] =
$$

$$
\begin{aligned}
&= \mathbb{E}\left[(1.1)^{\frac{32}{12} 2} \left\{ X \wedge \left(\frac{7,500,000}{(1.1)^{\frac{32}{12}}} \right) \right\}^2 \mid X \geq \frac{500,000}{(1.1)^{\frac{32}{12}}} \right] \\
&= (1.1)^{\frac{32}{12} 2} \, \mathbb{E}\left[\left\{ X \wedge \left(\frac{7,500,000}{(1.1)^{\frac{32}{12}}} \right) \right\}^2 \mid X \geq \frac{500,000}{(1.1)^{\frac{32}{12}}} \right] \text{, so by surf. interp. with weight } 2x \\
&= (1.1)^{\frac{32}{12} 2} \Bigg\{ \left(\frac{500,000}{(1.1)^{\frac{32}{12}}} \right)^2 \\
&\quad + \frac{1}{\mathbb{P}\left[X \geq \frac{500,000}{(1.1)^{\frac{32}{12}}} \right]} \left(\mathbb{E}\left[\left\{ X \wedge \left(\frac{7,500,000}{(1.1)^{\frac{32}{12}}} \right) \right\}^2 \right] - \mathbb{E}\left[\left\{ X \wedge \left(\frac{500,000}{(1.1)^{\frac{32}{12}}} \right) \right\}^2 \right] \right) \Bigg\} \\
&= 500,000^2 + \frac{(1.1)^{\frac{32}{12} 2}}{\Gamma(3; \frac{(1.1)^{\frac{32}{12}} \theta}{500,000})} \Bigg\{ \frac{\theta^2 \Gamma(1)}{\Gamma(3)} \left[1 - \Gamma(1; \frac{(1.1)^{\frac{32}{12}} \theta}{7,500,000}) \right] + \left(\frac{7,500,000}{(1.1)^{\frac{32}{12}}} \right)^2 \Gamma(3; \frac{(1.1)^{\frac{32}{12}} \theta}{7,500,000}) \\
&\quad - \frac{\theta^2 \Gamma(1)}{\Gamma(3)} \left[1 - \Gamma(1; \frac{(1.1)^{\frac{32}{12}} \theta}{500,000}) \right] - \left(\frac{500,000}{(1.1)^{\frac{32}{12}}} \right)^2 \Gamma(3; \frac{(1.1)^{\frac{32}{12}} \theta}{500,000}) \Bigg\} \text{ from App. A} \\
&= 500,000^2 + \frac{(1.1)^{\frac{32}{12} 2}}{\Gamma(3; 10 (1.1)^{\frac{32}{12}})} \Bigg\{ \frac{5,000,000^2}{2} \left[1 - \Gamma(1; \frac{2}{3} (1.1)^{\frac{32}{12}}) \right] + \left(\frac{7,500,000}{(1.1)^{\frac{32}{12}}} \right)^2 \Gamma(3; \frac{2}{3} (1.1)^{\frac{32}{12}}) \\
&\quad - \frac{5,000,000^2}{2} \left[1 - \Gamma(1; 10 (1.1)^{\frac{32}{12}}) \right] - \left(\frac{500,000}{(1.1)^{\frac{32}{12}}} \right)^2 \Gamma(3; 10 (1.1)^{\frac{32}{12}}) \Bigg\}
\end{aligned}
$$

using Theorem A.1 gives

$$= \quad 500{,}000^2 + \frac{(1.1)^{\frac{32}{12}2}}{1 - e^{-10(1.1)^{\frac{32}{12}}} \sum_{j=0}^{2} \frac{\left(10(1.1)^{\frac{32}{12}}\right)^j}{j!}} \left\{ \frac{5{,}000{,}000^2}{2} e^{-\frac{2}{3}(1.1)^{\frac{32}{12}}} \right.$$

$$+ \left(\frac{7{,}500{,}000}{(1.1)^{\frac{32}{12}}}\right)^2 \left[1 - e^{-\frac{2}{3}(1.1)^{\frac{32}{12}}} \sum_{j=0}^{2} \frac{\left(\frac{2}{3}(1.1)^{\frac{32}{12}}\right)^j}{j!}\right]$$

$$\left. - \frac{5{,}000{,}000^2}{2} e^{-10(1.1)^{\frac{32}{12}}} - \left(\frac{500{,}000}{(1.1)^{\frac{32}{12}}}\right)^2 \left[1 - e^{-10(1.1)^{\frac{32}{12}}} \sum_{j=0}^{2} \frac{\left(10\,(1.1)^{\frac{32}{12}}\right)^j}{j!}\right] \right\}$$

$$= \quad 1.197105 \cdot 10^{13} \text{ for the } 2^{nd} \text{ moment. (Or get the } \Gamma\text{s from software)}$$

So the standard deviation is $\sqrt{1.197105 \cdot 10^{13} \text{ -}2{,}960{,}973.5^2} = 1{,}789{,}884$

4. Write down formulas for the mean and first four central moments of the compound random variable

   $S = X_1 + ... + X_N$ with the X_j being i.i.d. copies of the random variable X

   in terms of the mean and first four central moments of N and X.

   SOLUTION: $C_S(z) = C_N(C_X(z))$ so $\mu_S = \mu_N \mu_X$. Taking derivatives for the cumulants of S the Faá displays are

$$j=2: \left|\begin{array}{ccc} & \mathbf{1} & \mathbf{2} \\ \mathbf{2} & 2 & 0 \\ \mathbf{1} & 0 & 1 \end{array}\right| \quad j=3: \left|\begin{array}{cccc} & \mathbf{1} & \mathbf{2} & \mathbf{3} \\ \mathbf{3} & 3 & 0 & 0 \\ \mathbf{2} & 1 & 1 & 0 \\ \mathbf{1} & 0 & 0 & 1 \end{array}\right| \quad j=4: \left|\begin{array}{ccccc} & \mathbf{1} & \mathbf{2} & \mathbf{3} & \mathbf{4} \\ \mathbf{4} & 4 & 0 & 0 & 0 \\ \mathbf{3} & 2 & 1 & 0 & 0 \\ \mathbf{2} & 1 & 0 & 1 & 0 \\ \mathbf{2} & 0 & 2 & 0 & 0 \\ \mathbf{1} & 0 & 0 & 0 & 1 \end{array}\right| \text{ So,}$$

$\kappa_{S2} = \kappa_{N2}\kappa_{X1}^2 + \kappa_{N1}\kappa_{X2},$

$\kappa_{S3} = \kappa_{N3}\kappa_{X1}^3 + 3\kappa_{N2}\kappa_{X1}\kappa_{X2} + \kappa_{N1}\kappa_{X3},$

$\kappa_{S4} = \kappa_{N4}\kappa_{X1}^4 + 6\kappa_{N3}\kappa_{X1}^2\kappa_{X2} + 4\kappa_{N2}\kappa_{X1}\kappa_{X3} + 3\kappa_{N2}\kappa_{X2}^2 + \kappa_{N1}\kappa_{X4}$

The answer to question #1 above gives the general formula for central moments in terms of cumulants. Combining these formulas,

$\mu_{S2} = \kappa_{S2} = \mu_{N2}\mu_X^2 + \mu_N\mu_{X2},$

$\mu_{S3} = \kappa_{S3} = \mu_{N3}\mu_X^3 + 3\mu_{N2}\mu_X\mu_{X2} + \mu_N\mu_{X3},$

$$\begin{aligned} \mu_{S4} &= 3\kappa_{S2}^2 + \kappa_{S4} \\ &= 3(\mu_{N2}\mu_X^2 + \mu_N\mu_{X2})^2 \\ &+ \left(\mu_{N4} - 3\mu_{N2}^2\right)\mu_X^4 + 6\mu_{N3}\mu_X^2\mu_{X2} + 4\mu_{N2}\mu_X\mu_{X3} + 3\mu_N\mu_{X2}^2 \\ &+ \mu_N\left(\mu_{X4} - 3\mu_{X2}^2\right), \text{ which simplifies just a little to} \end{aligned}$$

$$\mu_{S4} = \mu_{N4}\mu_X^4 + 6\left(\mu_{N2}\mu_N + \mu_{N3}\right)\mu_X^2\mu_{X2} + 4\mu_{N2}\mu_X\mu_{X3} \\ +3\mu_N^2\mu_{X2}^2 + \mu_N\mu_{X4}$$

5. If N is a compound Poisson-Poisson random variable with parameters $\lambda = .25$ for the primary variable and with parameters $\lambda = 1$ for the secondary variable, and if X can be approximated by a negative binomial random variable with $\beta = 4$ and $r = .25$ then calculate numerical values for the first 5 probabilities $\mathbb{P}[S = 0]$, $\mathbb{P}[S = 1]$, ... ,$\mathbb{P}[S = 4]$ for the random variable

   $S = X_1 + ... + X_N$ with the X_j being i.i.d. copies of the random variable X

   Hint: Think it through before you start calculating this one, and do the calculation in the easiest order.

   SOLUTIONS: Let M be Poisson with parameter $\lambda = .25$ and K be Poisson with parameter with parameters $\lambda = 1$ so

$$N = K_1 + \ ... \ K_M$$

   and $P_N(z) = P_M(P_K(z))$. But we also know that $P_S(z) = P_N(P_X(z))$, so $P_S(z) = P_M(P_K(P_X(z)))$ and we could write

$$\begin{aligned} S &= T_1 + \ ... \ + T_M \text{ where} \\ T &= X_1 + \ ... \ + X_K \text{ and} \\ P_T(z) &= P_K(P_X(z)) \text{ so} \\ P_S(z) &= P_M(P_T(z)) \end{aligned}$$

   Now we can calculate $\mathbb{P}(T = j)$ easily using a Panjer recursion (because K is Poisson, hence $(a, b, 0)$) and from those probabilities then calculate $\mathbb{P}(S = j)$ easily using another Panjer recursion (because M is Poisson, hence $(a, b, 0)$). Note that $\mathbb{P}[X = 0] = (1 + 4)^{-.25}$,

   $\mathbb{P}[X = j] = \mathbb{P}[X = j - 1] \frac{(.25+j-1)4}{j(1+4)}$,

   $\mathbb{P}[T = 0] = P_T(0) = P_K(P_X(0)) = P_K(\mathbb{P}[X = 0]) = e^{\mathbb{P}[X=0]-1}$,

   $\mathbb{P}[T = j]$ is Panjer recursion with $a = 0$ and $b = 1$,

   $\mathbb{P}[S = 0] = P_S(0) = P_M(P_T(0)) = P_M(\mathbb{P}[T = 0]) = e^{.25(\mathbb{P}[T=0]-1)}$,

   and $\mathbb{P}[S = j]$ is Panjer recursion with $a = 0$ and $b = .25$

j	$\mathbb{P}[X = j]$	$\mathbb{P}[T = j]$	$\mathbb{P}[S = j]$
0	0.668740305	0.718018679	0.931932091
1	0.133748061	0.096033606	0.022374200
2	0.066874030	0.054438957	0.012951937
3	0.040124418	0.035518553	0.008581876
4	0.026080872	0.024624433	0.006025727

Note that if you tried to use the EXCEL function NEGBINOMDIST to calculate $\mathbb{P}[X = j]$ you get errors. The reason is that EXCEL is NOT using the same definition of negative binomial distribution that we have been using in the textbook and in the notes. See App. B.2.1.4. EXCEL, by contrast, truncates the value of r to an integer. This is not at all what we have done in class or in our textbook.

It is your responsibility to verify that any software that you use is working with the same definition that you want. Never rely on just the name in any probability or statistics work! There is too much variation.

(I didn 't do this on purpose. It never occurred to me that any of you wouldn't just calculate negative binomial using the $(a, b, 0)$ rule.)