

Euler - Lagrange Equation

To find extremal value (max, min, stationary) of

$$I[u] = \int_a^b F(x, u(x), u'(x)) dx \quad \text{subject to constraints}$$

$$\left\{ H_i[u] = \int_a^b G_i(x, u(x), u'(x)) dx = C_i \right\} \quad \text{where for at least}$$

$$\text{one } i, \quad \frac{d}{dx} \frac{\partial}{\partial u'} G_i(x, u(x), u'(x)) - \frac{\partial}{\partial u} G_i(x, u(x), u'(x)) \neq 0,$$

solve for $u(x)$ in the Euler-Lagrange differential equation

$$\frac{d}{dx} \left[\frac{\partial}{\partial u'} F(x, u(x), u'(x)) + \sum_i \lambda_i \frac{\partial}{\partial u'} G_i(x, u(x), u'(x)) \right] -$$

$$= \left(\frac{\partial}{\partial u} F(x, u(x), u'(x)) + \sum_i \lambda_i \frac{\partial}{\partial u} G_i(x, u(x), u'(x)) \right) = 0$$

where $\{\lambda_i\}$ are arbitrary constants and $u(x)$ also

solves the constraints $\{ H_i[u] = C_i \}$