Math 5621 Financial Math II
Spring 2015
Final Exam Solutions
May 1 to May 6, 2015

This is an open book take-home exam. You may consult any books, notes, websites or other printed material that you wish. Having so consulted then submit your own answers as written by you.

Do NOT under any circumstances consult with any other person. Do NOT under any circumstances cut and paste any material from another source electronically into your answer. Do NOT under any circumstances electronically copy and paste from a spreadsheet that was not created entirely by you. Failure to follow these rules will be grounds for a failing grade for the course.

Put your name on all papers submitted and please show all of your work so that I can see your reasoning. The ten questions will be equally weighted in the grading. Please return the completed exams by 5 PM Wednesday, May 6 to my mailbox in the department office, under my office door MSB408, or by email.

1. The Black-Scholes formula for the price of a put option is

$$p = e^{-rT} K \left[1 - \Phi(d_2)\right] - S \left[1 - \Phi(d_1)\right]$$

where d_1 and d_2 are expressions that you can evaluate. Once you know d_1 the value of $\Phi(d_1)$ can be obtained from a spreadsheet function of normal probability values (or a published table of them.) Presumably, then, $[1 - \Phi(d_1)]$ must be the probability of some event. Explain what that event is and why $[1 - \Phi(d_1)]$ is its probability.

Solution

$[1 - \Phi(d_1)]$ is not the probability of any event. $[1 - \Phi(d_1)]$ is the conditional risk-neutral expected value $\mathbb{E}_q\left[e^{-rT}\frac{S_T}{S} | S_T < K\right]$ mulitiplied by the risk-neutral probability $\mathbb{P}_q\left[S_T < K\right]$. It is just an accident of the mathematical form of the lognormal density function that this complicated expected value can be found in a table of probability values. (note: using concepts not covered in class, it is possible to identify $[1 - \Phi(d_1)]$ as the probability that $S_T < K$ under an alternative make-believe risk-neutral probability measure that corresponds to using S_t rather than a risk-free investment account to discount future cash flows, also known as "using S_t as the numeraire" .)

2. A stock has a dividend yield of 2% and the company pays 7.5% interest on its long term debt. The ROE based on beginning of year equity is 16%. The are 10 million shares outstanding. The market to book ratio

is 1.25 and the share price is \$40. The interest payments on the long term debt amount to \$2.50 per share. What is the maximum possible growth rate the company can finance without using any new external sources of financing of any kind?

Solution

We are looking for the internal growth rate:

$$
\begin{aligned}
g &= \frac{PB \cdot NI}{NA} \text{ using beginning of year } NA = BV + D \\
&= PB \cdot ROE \cdot \frac{BV}{BV+D} \text{ using } ROE \text{ on begining of year } BV \\
&= \frac{NI - DIV}{NI} \cdot ROE \cdot \frac{1}{1+\frac{D}{BV}} \\
&= \left(ROE - \frac{DIV}{BV}\right) \cdot \frac{1}{1+\frac{\frac{Int}{BV}}{\frac{Int}{D}}} \\
&= \left(ROE - \frac{d \cdot MV}{BV}\right) \cdot \frac{1}{1+\frac{\frac{Int}{MV}\frac{MV}{BV}}{\frac{Int}{D}}} \\
&= (.16 - (.02)\cdot(1.25)) \cdot \frac{1}{1+\frac{\frac{2.50}{40}(1.25)}{.075}} \\
&= .0661
\end{aligned}
$$

3. Assume your company has three classes of securities in its financing structure: \$500 million (market value) of senior perpetual debt with a market yield of 5%; \$4 billion (market value) of junior high yield (junk) perpetual debt with a market yield of 15%; and \$250 million (market value) of common equity with a market capitalization rate of 40%. Assume a corporate tax rate of 35% and that, because of the high proportion of junk financing, the tax authorities grant tax deductibility to only 1/3 of the interest on the high yield financing.

   (a) What is the firm's weighted average cost of capital (WACC)?

   Solution

   Since the given facts included a market capitalization rate we can compute the WACC directly from (15.19) in the text (generalized to include the junk debt) as $WACC = \frac{250}{4750}.40 + \frac{500}{4750}(1-.35).05 + \frac{4000}{4750}(1-\frac{1}{3}(.35)).15 = .13605$ *or about* 13.6%

   (b) What can you conclude (if anything) about the cost of capital for an all-equity firm with the same operating risks? If you answer "nothing" give reasons.

   Solution

This is like exercise 15.1 but with junk debt instead of the preferred stock in the exercise, i.e. it is like equations (15.1) through (15.11) in the text, adjusted for the presence of the junk bonds. Let J stand for the market value of the junk bonds and X the portion of its interest that is deductible *and then, following the solution manual for 15.1, the value of the levered firm is*

$$
\begin{aligned}
V_L &= V_U + \tau_c B + X\tau_c J \\
\textit{where } V_U &= \textit{value of the unlevered (all equity) firm,} \\
\textit{so } V_U &= V_L - \tau_c B - X\tau_c J \\
\textit{But } V_U &= \frac{\mathbb{E}\left[EBIT(1-T)\right](1-\tau_c)}{\rho} \\
\textit{where } EBIT(1-T) &= \textit{cash flow from operations (perpetual)} \\
\textit{and } \rho &= \textit{the cost of capital for the unlevered firm.} \\
\textit{Thus, } \rho &= \frac{\mathbb{E}\left[EBIT(1-T)\right](1-\tau_c)}{V_L - \tau_c B - X\tau_c J} \\
\textit{But } V_L &= \frac{\mathbb{E}\left[EBIT(1-T)\right](1-\tau_c)}{WAAC} \textit{ so} \\
\mathbb{E}\left[EBIT(1-T)\right](1-\tau_c) &= WAAC \cdot V_L \textit{ and so} \\
\rho &= \frac{WAAC \cdot V_L}{V_L - \tau_c B - X\tau_c J} \\
&= \frac{WACC}{1 - \tau_c \frac{B}{V_L} - X\tau_c \frac{J}{V_L}} \\
&= \frac{.13605}{1 - .35\frac{500}{4750} - \frac{1}{3}(.35)\frac{4000}{4750}} \\
&= .1573 \textit{ or about } 15.7\%
\end{aligned}
$$

This entire analysis uses Modigliani-Miller style assumptions except for the taxes. Thus if considerations involving (1) expected value of future loss of deductions on debt (beyond what's already assumed) (2) expected value of future financial distress or (3) expected value of financial flexibility are important (as they are in fact likely to be for such a highly levered firm), then we have overstated the cost of capital for an all equity firm. Nothing in the given facts allows us to estimate the amount of this overstatement.

4. If a portfolio is to be constructed out of only two stocks, A and B, with $\sigma_A = .15$, $\sigma_B = .4$, and $\rho_{AB} = .6$, and if the risk free rate $r_f = .02$ and the expected returns on A and B are $r_A = .06$ and $r_B = .15$:

   (a) What is the proportion of A and B in the optimal portfolio that can be constructed from the two?

   Solution *By the formula demonstrated in class for the solution to the problem of creating the highest Sharpe ratio from a portfolio of n* *assests*

$$\begin{aligned}
\mathbf{w} &= \frac{\boldsymbol{\sigma}^{-1}\left(\mathbf{r}-r_f\mathbf{1}\right)}{\mathbf{1}^T\boldsymbol{\sigma}^{-1}\left(\mathbf{r}-r_f\mathbf{1}\right)} \text{ where in this case} \\
\boldsymbol{\sigma} &= \begin{Bmatrix} \sigma_A^2 & \rho_{AB}\sigma_A\sigma_B \\ \rho_{AB}\sigma_A\sigma_B & \sigma_B^2 \end{Bmatrix} \text{ so} \\
\boldsymbol{\sigma}^{-1} &= \frac{1}{\sigma_A^2\sigma_B^2-\rho_{AB}^2\sigma_A^2\sigma_B^2}\begin{Bmatrix} \sigma_B^2 & -\rho_{AB}\sigma_A\sigma_B \\ -\rho_{AB}\sigma_A\sigma_B & \sigma_A^2 \end{Bmatrix} \\
\boldsymbol{\sigma}^{-1}\left(\mathbf{r}-r_f\mathbf{1}\right) &= \frac{1}{\sigma_A^2\sigma_B^2-\rho_{AB}^2\sigma_A^2\sigma_B^2}\begin{Bmatrix} \sigma_B^2\left(r_A-r_f\right)-\rho_{AB}\sigma_A\sigma_B\left(r_B-r_f\right) \\ -\rho_{AB}\sigma_A\sigma_B\left(r_A-r_f\right)+\sigma_A^2\left(r_B-r_f\right) \end{Bmatrix} \\
\mathbf{1}^T\boldsymbol{\sigma}^{-1}\left(\mathbf{r}-r_f\mathbf{1}\right) &= \frac{\sigma_B^2\left(r_A-r_f\right)+\sigma_A^2\left(r_B-r_f\right)-\rho_{AB}\sigma_A\sigma_B\left(r_B+r_A-2r_f\right)}{\sigma_A^2\sigma_B^2-\rho_{AB}^2\sigma_A^2\sigma_B^2} \\
\mathbf{w} &= \frac{1}{\sigma_B^2\left(r_A-r_f\right)+\sigma_A^2\left(r_B-r_f\right)-\rho_{AB}\sigma_A\sigma_B\left(r_B+r_A-2r_f\right)}\cdot \\
&\quad \begin{Bmatrix} \sigma_B^2\left(r_A-r_f\right)-\rho_{AB}\sigma_A\sigma_B\left(r_B-r_f\right) \\ -\rho_{AB}\sigma_A\sigma_B\left(r_A-r_f\right)+\sigma_A^2\left(r_B-r_f\right) \end{Bmatrix} \\
w_A &= \frac{\sigma_B^2\left(r_A-r_f\right)-\rho_{AB}\sigma_A\sigma_B\left(r_B-r_f\right)}{\sigma_B^2\left(r_A-r_f\right)+\sigma_A^2\left(r_B-r_f\right)-\rho_{AB}\sigma_A\sigma_B\left(r_B+r_A-2r_f\right)} \\
&= .53666 \\
w_B &= \frac{-\rho_{AB}\sigma_A\sigma_B\left(r_A-r_f\right)+\sigma_A^2\left(r_B-r_f\right)}{\sigma_B^2\left(r_A-r_f\right)+\sigma_A^2\left(r_B-r_f\right)-\rho_{AB}\sigma_A\sigma_B\left(r_B+r_A-2r_f\right)} \\
&= .46334
\end{aligned}$$

(b) If the expected return on the market and its standard deviation are $r_M = .095$ and $\sigma_M = .20$, would you prefer to hold just the market portfolio or to hold the portfolio that you constucted in part a.? Explain why.

Solution *The Sharpe Ratio for the market is* $\frac{.095-.02}{.2} = .375$. *The Sharpe Ratio for the constructed portfolio is* $\frac{.53666(.06)+.46334(.15)-.02}{\sqrt{(.53666)^2(.15)^2+(.46334)^2(.4)^2+2(.53666)(.46334)(.6)(.15)(.4)}} = .337$. *The market portfolio has the higher Sharpe Ratio and is preferable.*

5. Capital Asset Pricing Model (CAPM)

(a) What correlations ρ_{AM} and ρ_{BM} between the returns on stocks A and B and the returns on the market would make the facts given in question 4. consistent with CAPM?

Solution *According to CAPM,* $r_A - r_f = \rho_{AM}\frac{\sigma_A}{\sigma_M}\left(r_M - r_f\right)$ *so* $\rho_{AM} = \frac{\sigma_M(r_A-r_f)}{\sigma_A(r_M-r_f)} = \frac{.20(.06-.02)}{.15(.095-.02)} = .7111$*; similarly* $\rho_{BM} = \frac{.20(.15-.02)}{.40(.095-.02)} = .8667$.

(b) If both of those correlations instead were $\rho_{AM} = \rho_{BM} = 0.75$, what would CAPM predict the expected return to be on the portfolio you constructed in 4.a.?

Solution *According to CAPM the portfolio return would be*

$$\begin{aligned}
w_A r_A + w_B r_B &= w_A \left(r_f + \rho_{AM} \frac{\sigma_A}{\sigma_M} (r_M - r_f) \right) + w_B \left(r_f + \rho_{BM} \frac{\sigma_B}{\sigma_M} (r_M - r_f) \right) \\
&= (.53666) \left(.02 + .75 \frac{.15}{.20} (.095 - .02) \right) \\
&\quad + (.46334) \left(.02 + .75 \frac{.40}{.20} (.095 - .02) \right) \\
&= .094766
\end{aligned}$$

(c) In that case (5.b. above) is the portfolio you constucted in 4.a. still the optimal one that can be built from these two assets? Why or why not? (You can do an easy check on this, without going all through the optimization calculation again.)

Solution *If it is optimal, it should have the highest Sharpe Ratio. The Sharpe Ratio is* $\frac{.094766 - .02}{\sqrt{(.53666)^2(.15)^2 + (.46334)^2(.4)^2 + 2(.53666)(.46334)(.6)(.15)(.4)}} =$ .30851.Make a slight change in the weights to .54 *and* .46 *giving a Sharpe ratio of*

$$\begin{aligned}
& \frac{.54 \left(r_f + \rho_{AM} \frac{\sigma_A}{\sigma_M} (r_M - r_f) \right) + .46 \left(r_f + \rho_{BM} \frac{\sigma_B}{\sigma_M} (r_M - r_f) \right) - .02}{\sqrt{(.54)^2 (.15)^2 + (.46)^2 (.4)^2 + 2 (.54)(.46)(.6)(.15)(.4)}} \\
= & \frac{.54 \left(.02 + .75 \frac{.15}{.20} (.095 - .02) \right)}{\sqrt{(.54)^2 (.15)^2 + (.46)^2 (.4)^2 + 2 (.54)(.46)(.6)(.15)(.4)}} \\
& + \frac{+ .46 \left(.02 + .75 \frac{.40}{.20} (.095 - .02) \right) - .02}{\sqrt{(.54)^2 (.15)^2 + (.46)^2 (.4)^2 + 2 (.54)(.46)(.6)(.15)(.4)}} \\
= & .30867
\end{aligned}$$

which is higher, meaning that the original one could not have been optimal (note the importance of keeping a fair amount of accuracy if you want to reason this way).

6. With the following expected returns and covariance matrix what are the weights w_1,w_2, and w_3 of each of the three assets in the optimal portfolio assuming the risk free rate is .001? You don't have to prove your answer but you do have to show how you calculated it.

	$\mathbf{j} =$	$\mathbf{1}$	$\mathbf{2}$	$\mathbf{3}$
	$\mathbf{r}_j =$	.0076	.0673	.1480
	$\boldsymbol{\sigma}_{i,j} =$			
$\mathbf{i} =$	$\mathbf{1}$	.01	$-.009$	0
	$\mathbf{2}$	$-.009$	.03	.02
	$\mathbf{3}$	0	.02	.06

Solution

see class notes on CAPM

The weight vector will be $\mathbf{w} = \frac{\boldsymbol{\sigma}^{-1}(\mathbf{r}-r_f\mathbf{1})}{\mathbf{1}^T\boldsymbol{\sigma}^{-1}(\mathbf{r}-r_f\mathbf{1})}$

$$\boldsymbol{\sigma}^{-1} = \left\langle \begin{array}{ccc} .01 & -.009 & 0 \\ -.009 & .03 & .02 \\ 0 & .02 & .06 \end{array} \right\rangle^{-1} = \left\langle \begin{array}{ccc} 153.173 & 59.0810 & -19.6937 \\ 59.0810 & 65.6455 & -21.8818 \\ -19.6937 & -21.8818 & 23.9606 \end{array} \right\rangle$$

$$\boldsymbol{\sigma}^{-1}(\mathbf{r} - r_f\mathbf{1}) = \left\langle \begin{array}{ccc} 153.173 & 59.0810 & -19.6937 \\ 59.0810 & 65.6455 & -21.8818 \\ -19.6937 & -21.8818 & 23.9606 \end{array} \right\rangle \left\langle \begin{array}{c} .0066 \\ .0663 \\ .1470 \end{array} \right\rangle =$$

$$\left\langle \begin{array}{c} 2.0330 \\ 1.5256 \\ 1.9415 \end{array} \right\rangle$$

So the weights are $\left\langle \begin{array}{c} 2.0330 \\ 1.5256 \\ 1.9415 \end{array} \right\rangle \div (2.0330 + 1.5256 + 1.9415) = \left\langle \begin{array}{c} .3696 \\ .2774 \\ .3530 \end{array} \right\rangle$

7. A company has net assets with a market value of \$7,500,000 and a financial structure involving 40% debt. The company believes that its current optimal financial structure should inolve 55% debt. The company is considering a new project that requires an investment of \$2,375,000. The company believes that after taking on the project it will have an optimal capital structure requiring 50% debt. If the company's after tax $WACC$ is 15%, its marginal cost of new debt is 6% before tax, and its marginal tax rate is 40%, then what after tax rate of return does the project need to earn in order to to be acceptable, assuming that it will be financed optimally?

   Solution

   Use subscripts b for the company before the project, p for the project itself, and a for the company after the project.

   To be acceptable the project needs to earn $WACC_p = \rho_p \left(1 - .40\frac{\Delta B}{\Delta(S+B)}\right)$ by equation (15.12) where ρ_p is the cost of capital for the project if it used no debt and $\frac{\Delta B}{\Delta(S+B)}$ is the proportion of debt in the optimal project financing. Since the optimal financing before the project was $55\%(7,500) = 4,125$ debt and after the project $50\%(7,500 + 2,375) = 4,937.5$ debt, then the optimal debt for the project must be $4,937.5 - 4,125 = 812.5$. Then

$\frac{\Delta B}{\Delta(S+B)}$ for the project is $\frac{812.5}{2{,}375} = .3421$ so $WACC_p = \rho_p(1 - .40 \cdot .3421) = .8632\rho_p$. We still need to figure out ρ_p.

To do that we use the relationship $\rho_a = w_b\rho_b + w_p\rho_p$ where $w_b = \frac{7500}{7500+2375} = .7595$ and $w_p = \frac{2375}{7500+2375} = .2405$, and $\rho_b = \frac{WAAC_b}{(1-.40\cdot.40)} = \frac{.15}{.84} = .1786$ by (15.12) and the comments following it. The relationship comes from CAPM:

$$
\begin{aligned}
\beta_a &= w_b\beta_b + w_p\beta_p \text{ by linearity of covariances so} \\
\rho_a &= r_f + \beta_a(r_M - r_f) \\
&= (w_b + w_p) r_f + \left(w_b\beta_b + w_p\beta_p\right)(r_M - r_f) \\
&= w_b(r_f + \beta_b(r_M - r_f)) + w_p\left(r_f + \beta_p(r_M - r_f)\right) \\
&= w_b\rho_b + w_p\rho_p \text{ so we now know that} \\
\rho_p &= \frac{\rho_a - w_b\rho_b}{w_p} \\
&= \frac{\rho_a - .7595 \cdot .1786}{.2405} \\
&= \frac{\rho_a}{.2405} - .5640 \text{ so} \\
WACC_p &= .8632\rho_p \\
&= 3.5892\rho_a - .4868
\end{aligned}
$$

Now we only need to figure out what ρ_a is. For that use

$$
\begin{aligned}
\rho_a(1 - .40 \cdot .50) &= WACC_a \text{ at the optimal financial position} \\
&\quad \text{of 50\% debt} \\
&= .50k_a + .50 \cdot .60 \cdot .06 \\
&= .50(w_bk_b + w_pk_p) + .50 \cdot .60 \cdot .06 \text{ where } k_b \\
&\quad \text{and } k_p \text{ are at their optiimal financial positions} \\
&= .50\left(.7595\frac{WACC_b - .55 \cdot .60 \cdot .06}{.45} + \right. \\
&\quad \left. .2405\frac{WAAC_p - .3421 \cdot .60 \cdot .06}{.6579}\right) + .50 \cdot .60 \cdot .06 \\
&\quad \text{where } WACC_b \text{ and } WAAC_p \text{ are at optimal} \\
&= .50\left(.7595\frac{.1786(1 - .55 \cdot .40) - .55 \cdot .60 \cdot .06}{.45} + \right. \\
&\quad \left. .2405\frac{3.5892\rho_a - .4868 - .3421 \cdot .60 \cdot .06}{.6579}\right) + .50 \cdot .60 \cdot .06 \\
.8\rho_a &= .6560\rho_a + .0276 \\
\rho_a &= .1917 \\
WACC_p &= 3.5892\rho_a - .4868 \\
&= .2012 \text{ which is the answer to the question}
\end{aligned}
$$

8. A commodities trading firm has the following market value balance sheet (in millions of \$):

	ASSETS		LIABILITIES
short-term	50	short term	100
treasury bonds	200	short commodity positions	750
long commodity positions	750	equity	150
	1,000		1,000

The standard deviations and correlations between returns on the asset and liability holdings are:

$\sigma\,(sta) = .02$ $\rho\,(sta, tb) = 0$ $\rho\,(sta, lcp) = 0$ $\rho\,(sta, stl) = 0$ $\rho\,(sta, scp) = 0$

$\sigma\,(tb) = .02$ $\rho\,(tb, lcp) = .8$ $\rho\,(tb, stl) = 0$ $\rho\,(tb, scp) = -.8$

$\sigma\,(lcp) = .25$ $\rho\,(lcp, stl) = 0$ $\rho\,(lcp, scp) = -.7$

$\sigma\,(stl) = .02$ $\rho\,(stl, scp) = 0$

$\sigma\,(scp) = .35$

(a) What is the standard deviation of returns on equity?

Solution

By equation (5.32), dividing everything in the balance sheet by the equity 150 so that weights add up to 1, the variance of the equity return is $\mathbf{w}^T\boldsymbol{\sigma}\mathbf{w} =$

$$\left\langle \begin{array}{ccccc} .3333 & 1.3333 & 5 & -.6666 & -5 \end{array} \right\rangle$$

$$\left\langle \begin{array}{ccccc} .02^2 & 0 & 0 & 0 & 0 \\ 0 & .02^2 & .8(.02)(.25) & 0 & -.8(.02)(.35) \\ 0 & .8(.02)(.25) & .25^2 & 0 & -.7(.25)(.35) \\ 0 & 0 & 0 & .02^2 & 0 \\ 0 & -.8(.02)(.35) & -.7(.25)(.35) & 0 & .35^2 \end{array} \right\rangle \left\langle \begin{array}{c} .3333 \\ 1.3333 \\ 5 \\ -.6666 \\ -5 \end{array} \right\rangle$$

$= 7.8164$ and the standard deviation is $\sqrt{7.8164} = 2.796$

(b) Suppose the firm wants to hedge by taking a position in treasury futures. If the price for a futures contract is $V_{tf} = \$90,000$ for each $\$100,000$ treasury future contract and

$$\begin{aligned} \sigma\,(tf) &= .35 \\ \rho\,(tf, sta) &= 0 \\ \rho\,(tf, tb) &= .9 \\ \rho\,(tf, lcp) &= .5 \\ \rho\,(tf, stl) &= 0 \\ \rho\,(tf, scp) &= -.3 \end{aligned}$$

then should the treasury futures position be long or short? How many contracts should they buy or sell? How much is the standard deviation of equity reduced?

Solution

Equation (5.33) gives $N = -\frac{1}{.09(.35)}(200\,(.9)\,(.02) + 750\,(.5)\,(.25) - 750\,(-.3)\,(.35)) = -5,590$, a short position in futures contracts. These have value $-5,590(.09) = -503.1$. The effect on equity is 0, with cash increasing by 503.1 received in the short sale and the new short position being -503.1. Now the variance is (remembering to divide by 150 equity) $\left\langle\ 3.6873 \quad 1.3333 \quad 5 \quad -.6666 \quad -5 \quad -3.354\ \right\rangle$

$$\left\langle \begin{array}{cccccc} .02^2 & 0 & 0 & 0 & 0 & 0 \\ 0 & .02^2 & .8(.02)(.25) & 0 & -.8(.02)(.35) & .9\,(.02)\,(.35) \\ 0 & .8(.02)(.25) & .25^2 & 0 & -.7(.25)(.35) & .5\,(.25)\,(.35) \\ 0 & 0 & 0 & .02^2 & 0 & 0 \\ 0 & -.8(.02)(.35) & -.7(.25)(.35) & 0 & .35^2 & -.3\,(.35)\,(.35) \\ 0 & .9\,(.02)\,(.35) & .5\,(.25)\,(.35) & 0 & -.3\,(.35)\,(.35) & .35^2 \end{array} \right\rangle \left\langle \begin{array}{c} 3.6873 \\ 1.3333 \\ 5 \\ -.6666 \\ -5 \\ -3.354 \end{array} \right\rangle$$

$= 6.4435$ and the standard deviation is $\sqrt{6.4435} = 2.538$

9. Your nuclear research department just discovered a way to turn lead into gold. With the price of gold at \$1200 per ounce this week you are quite excited and are making plans. You've already learned, for example, that you'll need to plan on annual spending of 1% of the value of any gold you produce just to store it safely and insure it. It's going to take you 12 years and a lot of money to implement the nuclear technology before you get your first output of gold, however, so you need to make an assumption about the price of gold 12 years from now in order to evaluate whether to go ahead with the investment today. The best experts you can find tell you that in their opinion the price of gold has a beta of 0, will be flat for the next two years while the market digests the Fed's tapering plans, but then it will advance 10% a year for 3 years reflecting the inflation of the dollar that must come sooner or later, followed by a steady 5% annual increase thereafter. The annual risk free rate for a 12 year horizon is 2.75%. What is the present value today of an ounce of gold produced 12 years from now?

   Solution Always trust the market price more than any expert's opinion, unless you are in the business of speculating (outguessing the market). Here your business is gold production, not speculation, so trust the market price of gold. With storage and insurance costs of 1% of the value of the gold per year the market is telling you that one ounce of gold twelve years from now can be produced without fail by putting $\$(.99)^{-12}\,Price-per-ounce-today$ worth of gold into insured storage today. It is a replicating portfolio guaranteed to pay off for one ounce of gold in twelve years. So the present value today of an ounce of gold produced twelve years from now is $\$(.99)^{-12}\,Price-per-ounce-today = \$1.1281781 \times 1200 = \$1353.81$.

10. For years, a company has plowed back 60% of earnings while making a 20% return on equity and maintaining a 3% dividend yield. The debt ratio has

remained constant. The market has priced the shares as if the growth rate corresponding to this financial performance could continue forever. By what % and in what direction will the share price change if the company suddenly announces, in a complete surprise to the market, that is has no further opportunities for profitable growth beyond its current scale of operations, it now plans no further growth at all, and will begin to pay out all of its earnings as dividends every year?

Solution

Under the scenario described, all of the current $PVGO$, present value of growth opportunities per-share, will disappear from the stock price at the time of the surprise announcement. So we get a decline in price:

$$\begin{aligned}
-\frac{PVGO}{P} &= -\frac{1}{P}\left(P - \frac{eps}{k_S}\right) \\
&= -\frac{1}{P}\left(P - \frac{\frac{eps(1-PB)}{1-PB}}{d+g}\right) \\
&= -\frac{1}{P}\left(P - \frac{\frac{\text{div}}{1-PB}}{d+g}\right) \\
&= -\frac{1}{P}\left(P - \frac{\text{div}}{(1-PB)\,(d+PB\cdot ROE)}\right) \\
&= -\left(1 - \frac{d}{(1-PB)\,(d+PB\cdot ROE)}\right) \\
&= -\left(1 - \frac{.03}{(1-.60)\,(.03+.60\,(.20))}\right) \\
&= -.50 \\
&= 50\% \text{ price decline}
\end{aligned}$$