Math 5621 Financial Math II
Fall 2015
Final Exam
December 11 to December 16, 2015

This is an open book take-home exam. You may consult any books, notes, websites or other printed material that you wish. Having so consulted then submit your own answers as written by you.

Do NOT under any circumstances consult with any other person. Do NOT under any circumstances cut and paste any material from another source electronically into your answer. Do NOT under any circumstances electronically copy and paste from a spreadsheet that was not created entirely by you. Failure to follow these rules will be grounds for a failing grade for the course.

Put your name on all papers submitted and please show all of your work so that I can see your reasoning. The four questions will be equally weighted in the grading. Please return the completed exams by 5 PM Wednesday, December 16 to my mailbox in the department office, under my office door MSB408, or by email.

1. A commodities trading firm has the following market value balance sheet (in millions of \$):

	ASSETS			LIABILITIES
short-term	150	short term		150
treasury bonds	200	short commodity positions		650
long commodity positions	650	equity		200
	1,000			1,000

The standard deviations and correlations between returns on the asset and liability holdings are:

$\sigma(sta) = .02 \ \rho(sta, tb) = 0 \ \rho(sta, lcp) = 0 \ \rho(sta, stl) = 0 \ \rho(sta, scp) = 0$

$\sigma(tb) = .02 \ \rho(tb, lcp) = .8 \ \rho(tb, stl) = 0 \ \rho(tb, scp) = .8$

$\sigma(lcp) = .25 \ \rho(lcp, stl) = 0 \ \rho(lcp, scp) = -.7$

$\sigma(stl) = .02 \ \rho(stl, scp) = 0$

$\sigma(scp) = .35$

(a) What is the standard deviation of returns on equity?

(b) Suppose the firm wants to hedge by taking a position in treasury futures. If the price for a futures contract is $V_{tf} = \$90,000$ for each $\$100,000$ treasury future contract and

$\sigma(tf) = .35 \ \rho(tf, sta) = 0 \ \rho(tf, tb) = .9 \ \rho(tf, lcp) = .5 \ \rho(tf, stl) = 0 \ \rho(tf, scp) = -.3$

then should the treasury futures position be long or short? How many contracts should they buy or sell? How much is the standard deviation of equity reduced?

2. For simplicity's sake, your investment firm wants to consider the entire world market for investments to consist of (a) the entire American market (North and South) as a single asset (b) the entire European market as a single asset (c) the entire Asian market as a single asset. (They might be making a big mistake to ignore the African market, but that's the decision they made.) The risk-free rate is .0025 and the expected rates of return for each of the three assets and the covariance matrix among them are

$\mathbf{i} =$	(a)	(b)	(c)
$\mathbf{r}_i =$	.0440	.0122	.1052

$\boldsymbol{\sigma}_{i,j} =$			
$\mathbf{j} =$ (a)	.04	$-.003$	.048
(b)	$-.003$	.0225	.012
(c)	.048	.012	.16

What is the optimal portion of each world asset in your firm's investment portfolio?

3. This problem involves a European put option expiring in T years with strike price K on an asset whose value today is S_0. The risk free rate (continuously compounded) is r.

   (a) Write down the Black-Scholes formula for V_0 the value today of the European put option

   (b) Write down the N-stage binomial tree formula for V_0 the value today of the European put option.

   (c) Explain why the term involving S_0 in the Black-Scholes formula for the value today of the European put option is NOT multiplied by e^{-rT}.

4. Look at formula (14.18) on page 504 of the textbook for the value of an all-equity firm experiencing finite supernormal growth

   (a) Is the term

$$\frac{EBIT_1 (1 - \tau_c)}{k_u} \left\{ \frac{Kr - k_u K}{k_u - Kr} \left[1 - \left(\frac{1 + Kr}{1 + k_u} \right)^N \right] \right\}$$

   equal to the present value today of V_N, the present value at the horizon time N of all the free cash flow after the horizon time N?

   (b) Given a reason for or analysis of why or why not in part a.