Math 5621 Financial Math II
Fall 2014
Final Exam Solutions
December 5 to December 10, 2014

This is an open book take-home exam. You may consult any books, notes, websites or other printed material that you wish. Having so consulted then submit your own answers as written by you.

Do NOT under any circumstances consult with any other person. Do NOT under any circumstances cut and paste any material from another source electronically into your answer. Do NOT under any circumstances electronically copy and paste from a spreadsheet that was not created entirely by you. Failure to follow these rules will be grounds for a failing grade for the course.

Put your name on all papers submitted and please show all of your work so that I can see your reasoning. The six questions will be equally weighted in the grading. Please return the completed exams by 5 PM Wednesday, December 10 to my mailbox in the department office, under my office door MSB408, or by email.

1. For years, a company has plowed back 60% of earnings while making a 20% return on equity and maintaining a 3% dividend yield. The debt ratio has remained constant. The market has priced the shares as if the growth rate corresponding to this financial performance could continue forever. By what % and in what direction will the share price change if the company suddenly announces, in a complete surprise to the market, that is has no further opportunities for profitable growth beyond its current scale of operations, it now plans no further growth at all, and will begin to pay out all of its earnings as dividends every year?

Solution

Under the scenario described, all of the current $PVGO$, present value of growth opportunities per-share, will disappear from the stock price at the time of the surprise announcement. So we get a decline in price:

$$\begin{aligned}
-\frac{PVGO}{P} &= -\frac{1}{P}\left(P-\frac{eps}{k_S}\right) \\
&= -\frac{1}{P}\left(P-\frac{\frac{eps(1-PB)}{1-PB}}{d+g}\right) \\
&= -\frac{1}{P}\left(P-\frac{\frac{\text{div}}{1-PB}}{d+g}\right) \\
&= -\frac{1}{P}\left(P-\frac{\text{div}}{(1-PB)\,(d+PB\cdot ROE)}\right) \\
&= -\left(1-\frac{d}{(1-PB)\,(d+PB\cdot ROE)}\right) \\
&= -\left(1-\frac{.03}{(1-.60)\,(.03+.60\,(.20))}\right) \\
&= -.50 \\
&= 50\% \text{ price decline}
\end{aligned}$$

2. Suppose that the current price in the market for blank silicon wafers used as raw material for chip manufacturing is \$0.50 per wafer. Your engineering staff tell you that their best and most reliable consultants forecast that the price of blank silicon wafers will rise an average rate of 2% per year for the next 3 years, 6% per year for the following 2 years, and reach long run equilibrium at 3% per year thereafter forever. You think that the forecast makes a lot of sense. You expect to be using 400,000 blank silicon wafer per year in your manufacturing operation for each of the next 20 years. Assume that blank silicon wafers have a $\beta = 0$, that the risk free rate is 1% for the next three years and 3% thereafter forever, and that any excess stock of silicon wafers from year to year can be stored for a negligible cost. For each of the the next 20 years you have purchased a European call option expiring at the end of that year on 400,000 blank silicon wafers with a strike price of \$0.55 per wafer to hedge your exposure to a rise in price. For each of the next 20 years you have sold short a European put option expiring at the end of that year on 400,000 blank silicon wafers with a strike price of \$0.55 per wafer in order to help finance the call option purchase. What is the value *today* of your net position in all of these options?

Solution

Since the cost of storing silicon wafers is assumed to be negligible, a position (long or short) in silicon wafers can be treated like a position in any other underlying asset. Current market values are superior to any expert forecast as a measure of the present value today of any future position in a tradable underlying asset. That means we can use option pricing theory for this problem, with the current market value of silicon wafers as the present value today of any future position in silicon wafers.

Put-call parity gives directly the value of the options positions described:

$$\begin{aligned} & \text{(value of call today)} - \text{(value of put today)} \\ = \; & \text{(value of underlying today)} - \text{(present value at risk-free rate of future strike price)} \end{aligned}$$

We can add this up over each of next 20 year's options:

$$\begin{aligned} & \text{Position Value} \\ = \; & (20)(.50)(400,000) - \left(\frac{1}{1.01} + \frac{1}{1.01^2} + \frac{1}{1.01^3}(1 + \frac{1}{1.03} + \frac{1}{1.03^2} + ... + \frac{1}{1.03^{17}}) \right) (.55)(400,000) \\ = \; & 4,000,000 - \left(\frac{1 - \frac{1}{1.01^3}}{.01} + \frac{1}{1.01^3} \frac{1 - \frac{1}{1.03^{17}}}{.03} \right) (.55)(400,000) \\ = \; & \$541,624 \end{aligned}$$

3. The Black-Scholes formula for the price of a call option is

$$c = S\Phi(d_1) - e^{-rT} K\Phi(d_2)$$

where d_1 and d_2 are expressions that you can evaluate. Once you know d_1 the value of $\Phi(d_1)$ can be obtained from a spreadsheet function of normal probabiity values (or a published table of them.) Presumably, then, $\Phi(d_1)$ must be the probability of some event. Explain what that event is and why $\Phi(d_1)$ is its probability.

Solution

$\Phi(d_1)$ is not the probability of any event. It is the conditional expected value $\tilde{\mathbb{E}}\left[e^{-rT}\frac{S_T}{S}|S_T > K\right]$ mulitiplied by the probability $\tilde{\mathbb{P}}\left[S_T > K\right]$. It is just an accident of the mathematical form of the lognormal density function that this complicated expected value can be found in a table of probability values. (note: using concepts not covered in class, it is possible to identify $\Phi(d_1)$ as the probability that $S_T > K$ under an alternative make-believe risk-neutral probability measure that corresponds to using S_t rather than a risk-free investment account to discount future cash flows, also known as "using S_t as the numeraire" .)

4. Assume your company has three classes of securities in its financing structure: \$500 million (market value) of senior perpetual debt with a market yield of 5%; \$4 billion (market value) of junior high yield (junk) perpetual debt with a market yield of 15%; and \$250 million (market value) of common equity with a market capitalization rate of 40%. Assume a corporate tax rate of 35% and that, because of the high proportion of junk financing, the tax authorities grant tax deductibility to only 1/3 of the interest on the high yield financing.

(a) What is the firm's weighted average cost of capital (WACC)?

Solution

Since the given facts included a market capitalization rate we can compute the $WACC$ directly from (15.19) in the text (generalized to include the junk debt) as $WACC = \frac{250}{4750}.40 + \frac{500}{4750}(1-.35).05 + \frac{4000}{4750}(1-\frac{1}{3}(.35)).15 = .13605$ *or about* 13.6%

(b) What can you conclude (if anything) about the cost of capital for an all-equity firm with the same operating risks? If you answer "nothing" give reasons.

Solution

This is like exercise 15.1 but with junk debt instead of the preferred stock in the exercise, i.e. it is like equations (15.1) through (15.11) in the text, adjusted for the presence of the junk bonds. Let J stand for the market value of the junk bonds and X the portion of its interest that is deductible *and then, following the solution manual for 15.1, the value of the levered firm is*

$$\begin{aligned}
V_L &= V_U + \tau_c B + X\tau_c J \\
\textit{where } V_U &= \textit{value of the unlevered (all equity) firm,} \\
\textit{so } V_U &= V_L - \tau_c B - X\tau_c J \\
\textit{But } V_U &= \frac{\mathbb{E}\left[EBIT(1-T)\right](1-\tau_c)}{\rho} \\
\textit{where } EBIT(1-T) &= \textit{cash flow from operations (perpetual)} \\
\textit{and } \rho &= \textit{the cost of capital for the unlevered firm.} \\
\textit{Thus, } \rho &= \frac{\mathbb{E}\left[EBIT(1-T)\right](1-\tau_c)}{V_L - \tau_c B - X\tau_c J} \\
\textit{But } V_L &= \frac{\mathbb{E}\left[EBIT(1-T)\right](1-\tau_c)}{WAAC} \textit{ so} \\
\mathbb{E}\left[EBIT(1-T)\right](1-\tau_c) &= WAAC \cdot V_L \textit{ and so} \\
\rho &= \frac{WAAC \cdot V_L}{V_L - \tau_c B - X\tau_c J} \\
&= \frac{WACC}{1 - \tau_c \frac{B}{V_L} - X\tau_c \frac{J}{V_L}} \\
&= \frac{.13605}{1 - .35\frac{500}{4750} - \frac{1}{3}(.35)\frac{4000}{4750}} \\
&= .1573 \textit{ or about } 15.7\%
\end{aligned}$$

This entire analysis uses Modigliani-Miller style assumptions except for the taxes. Thus if considerations involving (1) expected value of future loss of deductions on debt (beyond what's already assumed) (2) expected value of future financial distress or (3) expected value *of financial flexibility are important (as they are in fact likely to be for such a highly levered firm), then we have overstated the cost of capital for an all equity firm. Nothing in the given facts allows us to estimate the amount of this overstatement.*

5. With the following expected returns and covariance matrix what are the weights w_1,w_2, and w_3 of each of the three assets in the optimal portfolio assuming the risk free rate is .001? You don't have to prove your answer but you do have to show how you calculated it.

	j =	**1**	**2**	**3**
	$\mathbf{r}_j$=	.0076	.0673	.1480
	$\boldsymbol{\sigma}_{i,j}$=			
i =	**1**	.01	−.009	0
	2	−.009	.03	.02
	3	0	.02	.06

Solution

see class notes on CAPM

The weight vector will be $\mathbf{w} = \frac{\boldsymbol{\sigma}^{-1}(\mathbf{r}-r_f\mathbf{1})}{\mathbf{1}^T\boldsymbol{\sigma}^{-1}(\mathbf{r}-r_f\mathbf{1})}$

$$\boldsymbol{\sigma}^{-1} = \left\langle \begin{array}{ccc} .01 & -.009 & 0 \\ -.009 & .03 & .02 \\ 0 & .02 & .06 \end{array} \right\rangle^{-1} = \left\langle \begin{array}{ccc} 153.173 & 59.0810 & -19.6937 \\ 59.0810 & 65.6455 & -21.8818 \\ -19.6937 & -21.8818 & 23.9606 \end{array} \right\rangle$$

$$\boldsymbol{\sigma}^{-1}(\mathbf{r} - r_f\mathbf{1}) = \left\langle \begin{array}{ccc} 153.173 & 59.0810 & -19.6937 \\ 59.0810 & 65.6455 & -21.8818 \\ -19.6937 & -21.8818 & 23.9606 \end{array} \right\rangle \left\langle \begin{array}{c} .0066 \\ .0663 \\ .1470 \end{array} \right\rangle =$$

$$\left\langle \begin{array}{c} 2.0330 \\ 1.5256 \\ 1.9415 \end{array} \right\rangle$$

So the weights are $\left\langle \begin{array}{c} 2.0330 \\ 1.5256 \\ 1.9415 \end{array} \right\rangle \div (2.0330 + 1.5256 + 1.9415) = \left\langle \begin{array}{c} .3696 \\ .2774 \\ .3530 \end{array} \right\rangle$

6. A commodities trading firm has the following market value balance sheet (in millions of \$):

	ASSETS		LIABILITIES
short-term	50	short term	100
treasury bonds	200	short commodity positions	750
long commodity positions	750	equity	150
	1,000		1,000

The standard deviations and correlations between returns on the asset and liability holdings are:

$\sigma\,(sta) = .02$ $\rho\,(sta, tb) = 0$ $\rho\,(sta, lcp) = 0$ $\rho\,(sta, stl) = 0$ $\rho\,(sta, scp) = 0$

$\sigma\,(tb) = .02$ $\rho\,(tb, lcp) = .8$ $\rho\,(tb, stl) = 0$ $\rho\,(tb, scp) = -.8$

$\sigma\,(lcp) = .25$ $\rho\,(lcp, stl) = 0$ $\rho\,(lcp, scp) = -.7$

$\sigma\,(stl) = .02$ $\rho\,(stl, scp) = 0$

$\sigma\,(scp) = .35$

(a) What is the standard deviation of returns on equity?

Solution

By equation (5.32), dividing everything in the balance sheet by the equity 150 so that weights add up to 1, the variance of the equity return is $\mathbf{w}^T \boldsymbol{\sigma} \mathbf{w} =$

$$\left\langle \begin{array}{ccccc} .3333 & 1.3333 & 5 & -.6666 & -5 \end{array} \right\rangle$$

$$\left\langle \begin{array}{ccccc} .02^2 & 0 & 0 & 0 & 0 \\ 0 & .02^2 & .8(.02)(.25) & 0 & -.8(.02)(.35) \\ 0 & .8(.02)(.25) & .25^2 & 0 & -.7(.25)(.35) \\ 0 & 0 & 0 & .02^2 & 0 \\ 0 & -.8(.02)(.35) & -.7(.25)(.35) & 0 & .35^2 \end{array} \right\rangle \left\langle \begin{array}{c} .3333 \\ 1.3333 \\ 5 \\ -.6666 \\ -5 \end{array} \right\rangle$$

$= 7.8164$ and the standard deviation is $\sqrt{7.8164} = 2.796$

(b) Suppose the firm wants to hedge by taking a position in treasury futures. If the price for a futures contract is $V_{tf} = \$90,000$ for each \$100,000 treasury future contract and

$$\begin{aligned} \sigma\,(tf) &= .35 \\ \rho\,(tf, sta) &= 0 \\ \rho\,(tf, tb) &= .9 \\ \rho\,(tf, lcp) &= .5 \\ \rho\,(tf, stl) &= 0 \\ \rho\,(tf, scp) &= -.3 \end{aligned}$$

then should the treasury futures position be long or short? How many contracts should they buy or sell? How much is the standard deviation of equity reduced?

Solution

Equation (5.33) gives $N = -\frac{1}{.09(.35)}\left(200\,(.9)\,(.02) + 750\,(.5)\,(.25) - 750\,(-.3)\,(.35)\right) = -5,590$, a short position in futures contracts. These have value $-5,590(.09) = -503.1$. The effect on equity is 0, with cash increasing by 503.1 received in the short sale and the new short position being -503.1. Now the variance is (remembering to divide by 150 equity) $\left\langle\begin{array}{cccccc} 3.6873 & 1.3333 & 5 & -.6666 & -5 & -3.354 \end{array}\right\rangle$

$$\left\langle\begin{array}{cccccc}
.02^2 & 0 & 0 & 0 & 0 & 0 \\
0 & .02^2 & .8(.02)(.25) & 0 & -.8(.02)(.35) & .9\,(.02)\,(.35) \\
0 & .8(.02)(.25) & .25^2 & 0 & -.7(.25)(.35) & .5\,(.25)\,(.35) \\
0 & 0 & 0 & .02^2 & 0 & 0 \\
0 & -.8(.02)(.35) & -.7(.25)(.35) & 0 & .35^2 & -.3\,(.35)\,(.35) \\
0 & .9\,(.02)\,(.35) & .5\,(.25)\,(.35) & 0 & -.3\,(.35)\,(.35) & .35^2
\end{array}\right\rangle
\left\langle\begin{array}{c}
3.6873 \\ 1.3333 \\ 5 \\ -.6666 \\ -5 \\ -3.354
\end{array}\right\rangle$$

$= 6.4435$ and the standard deviation is $\sqrt{6.4435} = 2.538$