Opportunity Cost of Capital and WACC

The *"net present value" rule* says to accept the right to the cash flow random variable $\bar{C}_1$ at time 1, with expected value C_1, in exchange for a definite cash investment C_0 at time 0 whenever $\frac{C_1}{1+r} > C_0$ where the rate r used to discount cash flow is the "opportunity cost of capital" associated with the cash flow random variable $\bar{C}_1$. This is the rate at which the financial market values the cash flow random variable $\bar{C}_1$. In other words, $r = \frac{C_1}{M_0} - 1$ where M_0 is what the market will pay for the random cash flow $\bar{C}_1$. Why is this "opportunity cost of capital" always the right rate at which to discount a random cash flow?

If you discount at a rate $r <$ opportunity cost of capital then, intuitively, you would be willing to spend *more* to get the right to the cash flow than you could just *buy* the right to the same cash flow for in the market. You would be willing to just give up the use for a year of cash equal to the difference, which is ridiculous. *Who would be willing to do that?*

Mathematically, if $r < \frac{C_1}{M_0} - 1$ then $M_0 < \frac{C_1}{(1+r)}$ and anytime $M_0 < C_0 < \frac{C_1}{(1+r)}$ you would be willing to spend C_0 in order to get the right to $\bar{C}_1$. But you could buy the right to $\bar{C}_1$ in the market by paying M_0, so you are *giving up* $C_0 - M_0$ in *extra cash* for a year while winding up at the *same place* at the end of the year, namely, having the right to $\bar{C}_1$ in cash.

Who would be willing to give up free cash for a year?

If you discount at a rate $r >$ opportunity cost of capital then, intuitively, you would be willing to pass up the chance to spend *less* to get the right to the cash flow than you just could *finance the purchase* of the right to the same cash flow for in the market. You would be willing to just give up the use for a year of cash equal to the difference, which is ridiculous. *Who would be willing to do that?*

Mathematically, if $r > \frac{C_1}{M_0} - 1$ then $M_0 > \frac{C_1}{(1+r)}$ and anytime $M_0 > C_0 > \frac{C_1}{(1+r)}$ you would willing to pass up the chance to spend C_0 in order to get the right to $\bar{C}_1$. But you could get M_0 in the market in exchange for the right to $\bar{C}_1$, so you are *giving up* $M_0 - C_0$ in *extra cash* for a year while winding up at the *same* place at the end of the year, namely, having your original C_0 still available to you.

Who would be willing to give up free cash for a year?

Either way, you run the risk of making a mistake, *of giving up free cash for a year*, by discounting at a rate other than the opportunity cost of capital.

WACC: A company's *"weighted average cost of capital" (WACC)* often is taken as a convenient approximation for the opportunity cost of capital in financial work. (a) The *WACC* can be viewed as a kind of internal opportunity cost of capital: the alternative to investing in the project is the opportunity to reduce capital by the amount of the investment, saving the financing costs of that capital at the $WACC$ rate. (There can be an error in this line of thought.) (b) If the project at hand is "typical" for the company, then the $WACC$ reflects both market judgment and the effect of taxes. (This reasoning is safer, but the assumption that the project is "typical" doesn't always apply.)