Math 5621 Financial Math II
Spring 2014
Final Exam
May 2 to May 7, 2014

This is an open book take-home exam. You may consult any books, notes, websites or other printed material that you wish. Having so consulted then submit your own answers as written by you.

Do NOT under any circumstances consult with any other person. Do NOT under any circumstances cut and paste any material from another source electronically into your answer. Do NOT under any circumstances electronically copy and paste from a spreadsheet that was not created entirely by you. Failure to follow these rules will be grounds for a failing grade for the course.

Put your name on all papers submitted and please show all of your work so that I can see your reasoning. The ten questions will be equally weighted in the grading. Please return the completed exams by 5 PM Wednesday, May 7 to my mailbox in the department office, under my office door MSB408, or by email.

1. A commodities trading firm has the following market value balance sheet (in millions of $):

ASSETS		LIABILITIES	
short-term	50	short term	100
treasury bonds	200	short commodity positions	750
long commodity positions	750	equity	150
	1,000		1,000

The standard deviations and correlations between returns on the asset and liability holdings are:

$\sigma(sta) = .02$ $\rho(sta, tb) = 0$ $\rho(sta, lcp) = 0$ $\rho(sta, stl) = 0$ $\rho(sta, scp) = 0$

$\sigma(tb) = .02$ $\rho(tb, lcp) = .8$ $\rho(tb, stl) = 0$ $\rho(tb, scp) = .8$

$\sigma(lcp) = .25$ $\rho(lcp, stl) = 0$ $\rho(lcp, scp) = -.7$

$\sigma(stl) = .02$ $\rho(stl, scp) = 0$

$\sigma(scp) = .35$

(a) What is the standard deviation of returns on equity?

(b) Suppose the firm wants to hedge by taking a position in treasury futures. If the price for a futures contract is $V_{tf} = \$90,000$ for each $\$100,000$ treasury future contract and

$\sigma(tf) = .35$ $\rho(tf, sta) = 0$ $\rho(tf, tb) = .9$ $\rho(tf, lcp) = .5$ $\rho(tf, stl) = 0$ $\rho(tf, scp) = -.3$

then should the treasury futures position be long or short? How many contracts should they buy or sell? How much is the standard deviation of equity reduced?

2. Consider a put option with an exercise price of 18, expiring three years from today, on an underlying asset which pays no dividends, has a value of 15 today, and a standard deviation of annual return equal to .50. Use a binomial model with $N = 6$ steps and probabilities $q_u = q_d = \frac{1}{2}$ at each step. (Do **NOT** use a binomial model using the formulas in the textbook.) Use a risk-free annual rate of return of 0.5% for a three-year horizon.

   (a) What would be wrong with using u and d determined by the formulas in the textbook, given the other requirements in this question?

   (b) What is the value of the put option today if it is an American put option?

   (c) Logically, why is the value in (b) greater than 3, the amount I could realize by exercising the option immediately?

   (d) What is the first time that it might possibly be optimal to exercise this American put option, according to this binomial model?

   (e) At time $t = .5$, if you are at the up node of the tree will the value of the risk-free bonds in the replicating portfolio for a put option, after rebalancing the portfolio, be larger for an American put option or for a European put option? By how much?

   (f) Logically, why is the value of the risk-free bonds in the replicating portfolio in (e) larger for whichever option you chose in the answer?

3. With the following expected returns and covariance matrix what are the weights w_1,w_2, and w_3 of each of the three assets in the optimal portfolio assuming the risk free rate is .01? You don't have to prove your answer but you do have to show how you calculated it.

$j =$	1	2	3
$r_j =$	.0092	.0291	.0560

$\sigma_{i,j} =$			
$i =$ **1**	.01	−.009	0
2	−.009	.03	.02
3	0	.02	.06

4. Given the following covariance matrix

$j =$	1	2
$i =$ **1**	.04	−.06
2	−.06	.09

calculate the covariance between a portfolio that has 35% in asset 1 and 65% in asset 2 and another portfolio that has 75% in asset 1 and 25% in asset 2.

2

5. Alpha Gaming has a current price of $4 per share. You believe that the appropriate market capitalization rate for Alpha is 12%. Its annual sales are $2,400,000,000. Total annual expenses including depreciation, amortization, interest, and taxes are $2,100,000,000. On a book value basis debt is $720,000,000. The payout ratio is 75%. The price/book ratio is 200%. There are 400 million shares outstanding. (a) What present value of growth opportunities (PVGO) is implied by Alpha's market valuation? (b) Grow or Die: what is the maximum possible growth rate Alpha Gaming can attain without raising any new capital from outside the company?

6. A company has net assets with a market value of $7,500,000 and a financial structure involving 40% debt. The company believes that its current optimal financial structure should inolve 55% debt. The company is considering a new project that requires an investment of $2,375,000. The company believes that after taking on the project it will have an optimal capital structure requiring 50% debt. If the company's after tax $WACC$ is 15%, its marginal cost of new debt is 6% before tax, and its marginal tax rate is 40%, then what after tax rate of return does the project need to earn in order to to be acceptable, assuming that it will be financed optimally?

7. With a WAAC or Opportunity Cost of Capital of 17.5% (a) is a project with the following cash flows financially acceptable? (b) Is it acceptable to your boss who (irrationally) won't accept "any project with payouts that have less than a 20% return"? In justifying your answer, be sure to calculate (c) the Net Present Value (d) the IRR and (e) one other measure of the rate of return that helps you to answer (a) and (b). Finally, (f) be sure to explain to your boss why your answer to (b) fits his rule about 20%

t	CF_t
0	-2
1	0
2	1
3	-10
4	3
5	4
6	5
7	5
8	5
9	5
10	-6

8. Assume your company has three classes of securities in its financing structure: $500 million (market value) of senior perpetual debt with a market yield of 8%; $4 billion (market value) of junior high yield (junk) perpetual debt with a market yield of 22%; and $250 million (market value) of common equity with a market capitalization rate of 40%. Assume a corporate tax rate of 40% and that, because of the high proportion of junk financing, the tax authorities grant tax deductibility to only 35% of the interest on the high yield financing.

   (a) What is the firm's weighted average cost of capital (WACC)?

   (b) What can you conclude (if anything) about the cost of capital for an all-equity firm with the same operating risks? If you answer "nothing" give reasons.

9. Your nuclear research department just discovered a way to turn lead into gold. With the price of gold at $1200 per ounce this week you are quite excited and are making plans. You've already learned, for example, that you'll need to plan on annual spending of 1% of the value of any gold you produce just to store it safely and insure it. It's going to take you 12 years and a lot of money to implement the nuclear technology before you get your first output of gold, however, so you need to make an assumption about the price of gold 12 years from now in order to evaluate whether to go ahead with the investment today. The best experts you can find tell you that in their opinion the price of gold has a beta of 0, will be flat for the next two years while the market digests the Fed's tapering plans, but then it will advance 10% a year for 3 years reflecting the inflation of the dollar that must come sooner or later, followed by a steady 5% annual increase thereafter. The annual risk free rate for a 12 year horizon is 2.75%. What is the present value today of an ounce of gold produced 12 years from now?

10. The Black-Scholes formula for the price of a call option is

$$c = S\Phi(d_1) - e^{-rT}K\Phi(d_2)$$

where d_1 and d_2 are expressions that you can evaluate. Once you know d_1 the value of $\Phi(d_1)$ can be obtained from a spreadsheet function of normal probabiity values (or a published table of them.) Presumably, then, $\Phi(d_1)$ must be the probability of some event. Explain what that event is and why $\Phi(d_1)$ is its probability.