Math 5621 Financial Math II
Fall 2012
Final Exam Solutions
December 7 to December 12, 2012

This is an open book take-home exam. You may consult any books, notes, websites or other printed material that you wish. Having so consulted then submit your own answers as written by you.

Do NOT under any circumstances consult with any other person. Do NOT under any circumstances cut and paste any material from another source electronically into your answer. Do NOT under any circumstances electronically copy from a spreadsheet that was not created by you. Failure to follow these rules will be grounds for a failing grade for the course.

Put your name on all papers submitted and please show all of your work so that I can see your reasoning. The eight questions will be equally weighted in the grading. Please return the completed exams by 5 PM Wednesday, December 12 to my mailbox in the department office, under my office door MSB408, or by email.

1. A company is financed 100% equity with a cost of capital of 18%. It has an effective marginal tax rate of 42%. It decides to restructure its capital to 40% debt and 60% equity (on a market value basis) and finds that the market rate on its debt at that level is 10%. What are the after-tax WAAC and the cost of equity after the restructuring? If you need to make simplifying assumptions, do so but say exactly what they are.

   SOLUTIONS

   Using formiula (15.12) $WACC = .18\,(1 - .40 \cdot .42) = .14976$. The simplifying assumptions are (a) the only effect of debt is the tax deductibility of interest and (b) the 40/60 debt/equity ratio equals the company's long-range target.

   Using (15.18) or solving $WACC = .60k_S + .40(1 - .42).10$,

   $k_S = .18 + (1 - .42)(.18 - .10)\frac{.40}{.60} = .2109$

   or $k_S = \frac{.14976 - .40(1 - .42).1}{.6} = .2109$. The simplifying assumptions are the same.

2. Your nuclear research department just discovered a way to turn lead into gold. With the price of gold at $1700 per ounce this week you are quite excited and are making plans. You've already learned, for example, that you'll need to plan on annual spending of 1% of the value of any gold you produce just to store it safely and insure it. It's going to take you 12 years and a lot of money to implement the nuclear technology before you get your first output of gold, however, so you need to make an assumption about the price of gold 12 years from now in order to evaluate whether to go ahead with the investment. Best expert opinion is that the price of gold

has a beta of 0, will be flat for the next two years while the market digests its recent run-up, but then it will advance 10% a year for 3 years reflecting the inflation of the dollar that must come sooner or later, followed by a steady 5% annual increase thereafter. The risk free rate for a 12 year horizon is 1.75%. What is the present value today of an ounce of gold produced 12 years from now?

SOLUTION

Always trust the market price more than any expert's opinion, unless you are in the business of speculating (outguessing the market). Here your business is gold production, not speculation, so trust the market price of gold. With storage and insurance costs of 1% per year the market is telling you that \$1 worth of gold twelve years from now can be produced without fail by putting $\$(.99)^{-12}$ =\$1.1281781 worth of gold into insured storage today. In other words, \$1.1281781 is the value today of a replicating portfolio to guarantee an amount of gold twelve years from now that would have a value of \$1 today. So the present value today of an ounce of gold produced twelve years from now is $\$1700 \times 1.1281781$ =\$1917.90.

3. Consider a put option with an exercise price of 25, expiring two years from today, on an underlying asset which pays no dividends, has a value of 20 today, and a standard deviation of annual return equal to .40. Use a binomial model with $N = 8$ steps and probabilities $q_u = q_d = \frac{1}{2}$ at each step. (Do **NOT** use a binomial model with u and d determined by the formulas in the textbook.) Use a risk-free annual rate of return of 0.25% for a two-year horizon.

   (a) What would be wrong with using u and d determined by the formulas in the textbook, given the other requirements in this question?

   (b) What is the value of the put option today if it is an American put option?

   (c) Logically, why is the value in **(b)** greater than 5, the amount I could realize by exercising the option immediately?

   (d) What is the first time that it might possibly be optimal to exercise this American put option, according to this binomial model?

   (e) At time $t = .5$, if you are at the up-then-down node of the tree will the value of the risk-free bonds in the replicating portfolio for a put option, after rebalancing the portfolio, be larger for an American put option or for a European put option? By how much?

   (f) Logically, why is the value of the risk-free bonds in the replicating portfolio in **(e)** larger for whichever option you chose in the answer?

SOLUTION

(a) the textbook gives u and d values on the assumption that $ud = 1$ and the resulting $q_u = \frac{e^{r\frac{T}{N}} - d}{u-d}$. Here we are assuming that $q_u = \frac{1}{2}$ which does not correspond to $ud = 1$.

(b) 7.684723 see spreadsheet

(c) If I exercise today I eliminate the possibility of exercising for an even greater amount in the future should the the price drop. In other words, if I don't exercise I keep some choices open and choices (options) have value. The tree indicates that the value of the choices exceeds the amount available from exercising today.

(d) .75 years from now, see spreadsheet

(e) Larger for American option by 0.049089 see spreadsheet

(f) This is tricky: (1) The value of the option is higher for the American because there are more options (choices) in the future with the American, and options (choices) have value. (2) The short position in the underlying is larger for the American because the Δ is larger in absolute value (because the value of the future options in the American increase with lower values of the underlying, making the difference in the two possible future put values larger, increasing the Δ compared to the European). (3) The value of the risk free bonds is the option value plus the absolute value of the short position, so (1) and (2) force it to be larger for the American.

4. The Black-Scholes formula for the price of a call option is

$$c = S\Phi(d_1) - e^{-rT}K\Phi(d_2)$$

where d_1 and d_2 are expressions that you can evaluate. Once you know d_1 the value of $\Phi(d_1)$ can be obtained from a spreadsheet function of normal probabiity values (or a published table of them.) Presumably, then, $\Phi(d_1)$ must be the probability of some event. Explain what that event is and why $\Phi(d_1)$ is its probability.

SOLUTION

$\Phi(d_1)$ is not the probability of any event. It is the conditional expected value $\mathbb{E}\left[e^{-rT}\frac{S_T}{S}|S_T > K\right]$. It is just an accident of the mathematical form of the lognormal density function that this complicated expected value can be found in a table of probability values. (note: using concepts not covered in class, it is possible to identify $\Phi(d_1)$ as the probability that $S_T > K$ under an alternative make-believe risk-neutral probability measure that corresponds to using S_t rather than a risk-free investment account to discount future cash flows, also known as "using S_t as the numeraire" .)

5. Consider the situation of exercise 5.14 in the textbook. If the expected

returns on each balance sheet category are as follows:

short term assets 0.2%
U.S. Treasury bonds 2%
loans 5%
short term liabilities 1%
deposits 0.2%

what is the Sharpe ratio of the equity holders' position before and after taking the recommended T-Bond futures position as a hedge? If you need to make assumptions specify clearly what you are assuming.

SOLUTION

To get the Sharpe ratio you need to assume a risk-free rate. I assumed .002 considering the given returns on short term assets and liabilities. Some of you assumed .02 based on the given long term government bond yields. As long as you specified what you were assuming you got full credit. For a bank anaysis, my shorter term rate is probably what would actually be used in practice. Using my assumption, and taking the standard deviation from the solution manual, before the hedge:

$$\begin{aligned}\text{Sharpe Ratio} &= \frac{\frac{100}{100}.002 + \frac{200}{100}.02 + \frac{700}{100}.05 - \frac{50}{100}.01 - \frac{850}{100}.002 - .002}{.574482} \\ &= .640577\end{aligned}$$

But there is a miscalculation in the solution manual. If you caught it you get extra credit. The correct value for the standard deviation is .559039 which produces here a Sharpe Ration of .6582725.

After the hedge, you need to be careful about two things: First, as indicated on the course website, the solution manual has an error for the after-hedge situation. In particular, taking a short position worth (\$300.6) million in the Treasury futures would generate \$300.6 million in cash, adding to the original \$100 million of short term assets for a total of \$400.6 million of short term assets, and leaving the net equity unchanged at \$100 million. Then in calculating the standard deviation as in the solution manual the vector w' should be $[4.006, 2.0, 7.0, (.5), (8.5), (3.006)]$ and w changes accordingly. The covariance matrix doesn't change so recalculating the standard deviation gives .510592 after the hedge, rather than the figure in the solutions manual. The second thing to be careful about is the return on the hedged position. You are not in a short position in Treasury bonds, but rather in Treasury bond futures. What is the return on that? Well, the logic of hedging is that you change your variance without (ideally) changing your return. So the best theoretical assumption is that the net hedged position, consisting of the \$(300.6) million in futures and the \$300.6 million in additional short term assets has a return of 0%.

That means that we should assume that the Treasury bond futures themselves have a return of .002, i.e. 0.2%. (In practice, you might assume slightly more so that the net hedged position carried a small cost. We'll ignore that here.) So the result is that after the hedge:

$$\begin{aligned} \text{Sharpe Ratio} &= \frac{\frac{400.6}{100}.002 + \frac{200}{100}.02 + \frac{700}{100}.05 - \frac{50}{100}.01 - \frac{850}{100}.002 - \frac{300.6}{100}.002 - .002}{.510592} \\ &= .720732 \end{aligned}$$

In practice, it would be wise to see how much of an unfavorable drift (actual expected return on the hedge different from the theoretical zero) you can withstand without reducing the original Sharpe ratio of .640577. Using the $300.6 million value for the hedge, an unfavorable drift of .013615 gives back the original Sharpe ratio:

$$\begin{aligned} \text{Sharpe Ratio} &= \frac{\frac{400.6}{100}.002 + \frac{200}{100}.02 + \frac{700}{100}.05 - \frac{50}{100}.01 - \frac{850}{100}.002 - \frac{300.6}{100}(.002 + .013615) - .002}{.510592} \\ &= .640577 \end{aligned}$$

Unfavorable drift in the hedge cost can arise from the expenses of managing the hedge, such as interest on margin requirements, dealer's spread, etc. These would be unlikely to be as large as 1.3615%, so the strategy is viable from a cost perspective. Since no hedge is perfect, the 1.3615% also gives you an idea of how much margin for error you can live with in the hedging and not have given up the benefit of hedging from a risk-reward perspective.

6. How would each of the following actions affect a firm's current ratio?

   SOLUTION

   Current Ratio is Current Assets/Current Liabilities (p.792 of the financial statements handout).

   (a) Sell inventory for cash - No Change, two parts of current assets get swapped

   (b) Borrow short term from a bank to pay a supplier- No Change, two parts of current liabilities get swapped

   (c) Collect an old bill from a customer that has been overdue for 2 years - Increase Current Ratio - cash goes up but receivables remains the same because a 2 year old receivable would have been written off already.

   (d) Buy more inventory for cash - No Change, two parts of current assets get swapped

7. Hannibal Inc., with a WAAC of 16.65%, is growing both its earnings and its dividends at 5.55% % per year. Assume that it can do that forever.

Scipio Inc., with a WACC of 7%, is growing both its earnings and its dividends at 3.33% per year. Assume it can do that forever. The two companies have exactly the same values for assets, earnings and dividends this year. Can you tell whether Hannibal's stock price or Scipio's stock price benefits more from the assumed growth rate? Why or why not? Explain your conclusion with specific formula(s). (There might be more than one correct explanation . . . you only need to give one.)

SOLUTION

$$\begin{aligned} PVGO + \frac{NI}{r} &= price = \frac{DIV}{r-g} \text{ so} \\ \frac{PVGO}{price} &= \frac{\frac{DIV}{r-g} - \frac{NI}{r}}{\frac{DIV}{r-g}} = 1 - \frac{(r-g)}{r}\frac{NI}{DIV} = 1 - (1 - \frac{g}{r})\frac{NI}{DIV} \end{aligned}$$

Hannibal: $g = .0555$, $r = .1665$, $\frac{g}{r} = .333$

Scipio: $g = .0333$, $r = .07$, $\frac{g}{r} = .476$.

Everything else is identical, so Scipio has a higher $PVGO$ % in its stock price.

8. A cell phone manufacturing plant that costs $400 million to build can produce a new line of voice recognition sets that will generate PV of future cash flow equal to $560 million if successful in the market, but only $200 million if market acceptance of the new gimmick is low. You believe that the probability of success is 50%. Would you build the plant? Would your decision change if you were certain that, if market acceptance turned out to be low, you could sell the plant to a competitor for an amount whose PV today is $250 million? Give quantitative reason for your answers. Why might you want to be suspicious about the $250 million assumptions? What sort of facts or reasons might alleviate your suspicions?

   SOLUTION

   $\mathbb{E}[PV(cash)] = .5(560) + .5(200) = 380 < 400$ so NPV is negative and I would not build the plant.

   But if you were certain about ability to sell the plant then

   $\mathbb{E}[PV(cash)] = .5(560) + .5(250) = 405 > 400$ so NPV is positive and if it was certain that I could sell the plant for 250 then I would change my decision and build the plant.

   I might be very suspiscious of that assumption however, because why would somebody buy something from me at a price in excess of its value tome? There must be something missing in the analysis.

   Some additional facts and reasons that might alleviate my suspicions about assuming the $250 million would be if the competitor has some capability that I do not have and can't reproduce cheaply that would make the plant more valuable to them than to me. For example, the competitor might have cheaper or more efficient labor, or more efficient marketing or logistics than me, enough so to make the net cash flow from owning the plant larger for them than for me. In other words, they might have a real option that I don't and part of their strike price for the option could be the plant that they buy from me.