Math 5621
Financial Mathematics II
Fall 2009
Final Examination
December 11-16, 2009

This is a take-home examination due back to me by 5 PM Wednesday, December 16, in my department mail box, under my office door or by email. You may consult any written source, including textbooks, notes, solution manuals, websites, or anything else written. You may not consult with any other person. Doing so will be cause for failing the course. Be sure to put your name on all papers submitted. Please show all of your work and give all reasoning and calculations (if any) associated with your answers. The four questions will be equally weighted in the grading.

1. Consider a put option with an exercise price of 25, expiring two years from today, on an underlying asset which pays no dividends, has a value of 20 today, and a standard deviation of annual return equal to .40. Use a binomial model with $N = 8$ steps and probabilities $q_u = q_d = \frac{1}{2}$ at each step. (Do **NOT** use a binomial model with u and d determined by the formulas in the textbook.) Use a risk-free annual rate of return of 2%.

   (a) What would be wrong with using u and d determined by the formulas in the textbook, given the other requirements in this question?

   (b) What is the value of the put option today if it is an American put option?

   (c) Logically, why is the value in (**b**) greater than 5, the amount I could realize by exercising the option immediately?

   (d) What is the first time that it might possibly be optimal to exercise this American put option, according to this binomial model?

   (e) At time $t = .5$, if you are at the up-then-down node of the tree will the value of the risk-free bonds in the replicating portfolio for a put option, after rebalancing the portfolio, be larger for an American put option or for a European put option? By how much?

   (f) Logically, why is the value of the risk-free bonds in the replicating portfolio in (**e**) larger for whichever option you chose in the answer?

2. Discuss the following two financial structure situations:

(a) Two large international airlines, **A** and **B,** are essentially identical in their markets, operating characteristics and future opportunities. Airline **A** has a debt to equity ratio (on a market value basis) of 3/2. Airline **B** has a debt to equity ratio (on a market value basis) of 1/4. Neither has a significant amount of cash or short term investments on the balance sheet.

   i. Which one is probably closer to the ideal capital structure for a large international airline?
   ii. Why? (Give very precise reasons, including a picture or an equation of some kind.)
   iii. Does this mean for sure that the other one, the one not so close to the ideal capital structure for a large international airline, is being mismanaged financially?
   iv. Why or why not? (Be very precise).
   v. Should **B** have paid much higher dividends in the past than it actually did? Why or why not?
   vi. Should **B** consider increasing its dividend now? Why or why not? If so, by a very large amount so as quickly to look more like **A** financially? Why or why not?

(b) Two large and growing software firms, **C** and **D**, are essentially identical in their markets, operating characteristics and future opportunities. Firm **C** has a debt to equity ratio (on a market value basis) of 0 and cash plus short term investments equal to 40% of the market value of equity. Firm **D** has a debt to equity ratio (on a market value basis) of 3/2 and cash plus short term investments equal to 10% of the market value of equity.

   i. Which one is probably closer to the ideal capital structure for a large and growing software firm?
   ii. Why? (Give very precise reasons, including a picture or an equation of some kind.)
   iii. Does this mean for sure that the other one, the one not so close to the ideal capital structure for a large and growing software firm, is being mismanaged financially?
   iv. Why or why not? (Be very precise).
   v. Should **C** have paid much higher dividends in the past than it actually did? Why or why not?
   vi. Should **C** consider increasing its dividend now? Why or why not? If so, by a very large amount so as quickly to look more like **D** financially? Why or why not?
   vii. Should **D** consider a plan to start reducing its debt? Why or why not? If so, should it start by using most of its current cash and short term investments to repay debt? Why or why not?

3. ABC Educational Technologies is considering a $2 million project that it believes will not change its operating risk characteristics. Currently, ABC's after-tax WACC is 9.4% with a 40% debt to net assets capital structure (which the company believes is optimal for the risks and opportunities the company faces) and a marginal cost of debt of 9% (before taxes). The marginal tax rate is 38.5%. If current net assets (before the project) are $8 million and the project will be financed with newly raised equity capital:

    (a) What rate of return must the project earn in order to be acceptable?

    (b) What rate of return would the project have had to earn in order to be acceptable if the company had decided to include an optimal level of debt in the newly raised capital to finance the project, defining optimal by tax and risk considerations?

    (c) Can you think of a reason why it might still have been optimal for the company to finance the project entirely with new equity capital despite the answers in (a) and (b)?

4. Give at least two completely different sets of assumptions, either one of which is sufficient to conclude that the market portfolio is an efficient portfolio?