Math 5621
Financial Mathematics II
Fall 2009
Final Examination Solutions
December 11-16, 2009

This is a take-home examination due back to me by 5 PM Wednesday, December 16, in my department mail box, under my office door or by email. You may consult any written source, including textbooks, notes, solution manuals, websites, or anything else written. You may not consult with any other person. Doing so will be cause for failing the course. Be sure to put your name on all papers submitted. Please show all of your work and give all reasoning and calculations (if any) associated with your answers. The four questions will be equally weighted in the grading.

1. Consider a put option with an exercise price of 25, expiring two years from today, on an underlying asset which pays no dividends, has a value of 20 today, and a standard deviation of annual return equal to .40. Use a binomial model with $N = 8$ steps and probabilities $q_u = q_d = \frac{1}{2}$ at each step. (Do **NOT** use a binomial model with u and d determined by the formulas in the textbook.) Use a risk-free annual rate of return of 2%.

   (a) What would be wrong with using u and d determined by the formulas in the textbook, given the other requirements in this question?

   u and d values from the textbook require use of probability values q_u and $q_d \neq \frac{1}{2}$

   (b) What is the value of the put option today if it is an American put option?

   7.313496286 (see spreadsheet)

   (c) Logically, why is the value in **(b)** greater than 5, the amount I could realize by exercising the option immediately?

   The expected value of the present value of exercising the option in the future is greater than 5, using the risk-free rate and make-believe risk-neutral probabilitities. But these give proper values today since a replicating portfolio can be constructed to assure future payoffs equal to the ones on the binomial tree.

   (d) What is the first time that it might possibly be optimal to exercise this American put option, according to this binomial model?

   $t = .75$ is the first time that the option value at some node of the tree equals the payoff on the option. (see spreadsheet)

   (e) At time $t = .5$, if you are at the up-then-down node of the tree will the value of the risk-free bonds in the replicating portfolio for a put

option, after rebalancing the portfolio, be larger for an American put option or for a European put option? By how much?

The American put option, by 0.630153616 (see spreadsheet)

(f) Logically, why is the value of the risk-free bonds in the replicating portfolio in **(e)** larger for whichever option you chose in the answer?

(1) The value of the option is higher for the American because there are more options (choices) in the future with the American, and options (choices) have value. (2) The short position in the underlying is larger for the American because the Δ is larger in absolute value (because the value of the future options in the American increase with lower values of the underlying, making the difference in the two possible future put values larger, increasing the Δ compared to the European). (3) The value of the risk free bonds is the option value plus the absolute value of the short position, so (1) and (2) force it to be larger for the American.

2. Discuss the following two financial structure situations:

(a) Two large international airlines, **A** and **B,** are essentially identical in their markets, operating characteristics and future opportunities. Airline **A** has a debt to equity ratio (on a market value basis) of 3/2. Airline **B** has a debt to equity ratio (on a market value basis) of 1/4. Neither has a significant amount of cash or short term investments on the balance sheet.

i. Which one is probably closer to the ideal capital structure for a large international airline?
A is closer to the ideal.

ii. Why? (Give very precise reasons, including a picture or an equation of some kind.)
With long lead times for buying airplanes and landing gate rights and for flight scheduling, airlines have little value in flexibility. With most capital invested in tangible things like airplanes and tangible rights like landing gate rights, both of which provide security for debt, the costs of financial distress and bankruptcy are reltively small for airlines. Therefore, both sources to push optimum value $(B + S)$ to the left are fairly small and the ideal capital structure for an airline includes large amounts of debt. **A** has 60% debt/net-assets while **B** has 20%. While precision about the ideal is impossible, clearly **A** is closer to the ideal for an airline. See figure 2a. in the figures file

iii. Does this mean for sure that the other one, the one not so close to the ideal capital structure for a large international airline, is being mismanaged financialy?
Not necessarily.

iv. Why or why not? (Be very precise).
If **B** has had a period of unusually high profitability in its recent history and has not yet had time for prudent dividends to pay out the unusual profits then the large resulting retained earnings would account for S being relatively larger than B

v. Should **B** have paid much higher dividends in the past than it actually did? Why or why not?
Maybe, if the larger retained earnings has been on the books for a long time. But if it is a more recent development, then it might not have been prudent for **B** to make a large dividend increase because then it might be forced to reduce the dividend (something to be desparately avoided) after the period of unusually high profitability comes to an end.

vi. Should **B** consider increasing its dividend now? Why or why not? If so, by a very large amount so as quickly to look more like **A** financially? Why or why not?
Perhaps some increase to begin reducing S relative to B. But not by a very large amount, certainly not more than projected (rather than past) profitability levels can support because once the desired capital structure has been reached it might be necessary reduce the dividend (something to be desparately avoided) if it has been set too large.

(b) Two large and growing software firms, **C** and **D**, are essentially identical in their markets, operating characteristics and future opportunities. Firm **C** has a debt to equity ratio (on a market value basis) of 0 and cash plus short term investments equal to 40% of the market value of equity. Firm **D** has a debt to equity ratio (on a market value basis) of 3/2 and cash plus short term investments equal to 10% of the market value of equity.

i. Which one is probably closer to the ideal capital structure for a large and growing software firm?
C is closer to the ideal.

ii. Why? (Give very precise reasons, including a picture or an equation of some kind.)
The bulk of the value of a software firm comes from the embedded real options it contains as a generator of future innovations. Furthermore, a software firm will have a huge erosion of value in the event of financial distress or bankruptcy because its biggest assets are (a) its employees, who walk out the door every evening and (b) its reputation with customers for being there with service and upgrades in the future. Therefore both forces to push optimum value $(B + S)$ to the left are relatively large and the optimum capital structure for a software firm contains little or no debt and lots of cash and liquid assets. See figure 2b. in the figures file.

iii. Does this mean for sure that the other one, the one not so close to the ideal capital structure for a large and growing software firm, is being mismanaged financially?
Not necessarily.

iv. Why or why not? (Be very precise).
D may have had (a) a run of very bad profitability for reasons other than financial management or (b) a recent opportunity to make a very large investment (such as an acquisition or a major expansion of operations) in order to exercise a real option it created in the past that moved in to the money. Either of those situations could have caused it to spend down its cash balances and to take on debt.

v. Should **C** have paid much higher dividends in the past than it actually did? Why or why not?
Almost surely not. By keeping its dividend lower it has maintained high cash/no debt which is ideal for software firm.

vi. Should **C** consider increasing its dividend now? Why or why not? If so, by a very large amount so as quickly to look more like **D** financially? Why or why not?
Almost surely not. By keeping its dividend low it will maintain high cash/no debt which is ideal for software firm. The only use it should make of cash is to make investments that exercise its real options if they should become in the money.

vii. Should **D** consider a plan to start reducing its debt? Why or why not? If so, should it start by using most of its current cash and short term investments to repay debt? Why or why not?
Probably, but only if it can be done without reducing the dividend (to be avoided almost always) and without sacrificing necessary investments to maintain the value of real options. If that can be done, then the lower debt would leave it in a better position to exercise more real options in the future. It should probably not reduce debt at the expense of current cash and short term investments, because it could face a need to spend cash to exercise a real option at a time when credit markets are tight and borrowing could be hard.

3. ABC Educational Technologies is considering a $2 million project that it believes will not change its operating risk characteristics. Currently, ABC's after-tax WACC is 9.4% with a 40% debt to net assets capital structure (which the company believes is optimal for the risks and opportunities the company faces) and a marginal cost of debt of 9% (before taxes). The marginal tax rate is 38.5%. If current net assets (before the project) are $8 million and the project will be financed with newly raised equity capital:

   (a) What rate of return must the project earn in order to be acceptable?

ANS: 9.7%

This is just like problem #15.7 so refer to the solution manual and to chapter 15. Since the the project will not change operating risk, it is reasonable to assume that the project could support a 40% debt to net assets capital structure. Using equation (15.13) $WACC = \rho\left(1 - \tau_C \frac{B}{B+S}\right)$ so with the given values, solving for ρ, we get $\rho = \frac{.094}{1-.385(.4)} = .1111$ for the all-equity required return. After taking on the project, the new debt to net asset ratio will be $\frac{.4(8)}{8+2} = .32$, so the required return can be found from (15.13), required return= $WACC = .1111(1 - .385(.32)) = .097$

(b) What rate of return would the project have had to earn in order to be acceptable if the company had decided to include an optimal level of debt in the newly raised capital to finance the project, defining optimal by tax and risk considerations? ANS: 9.4%

Since the project risk characteristics match the company's and company thinks that 40% is the optimal debt to net assets capital structure, then 40% is the optimal level of debt in financing the new project. So, after taking on the project the new debt to net asset ratio will still be 40%, so the required return can be found from (15.13), required return= $WACC = .1111(1 - .385(.40)) = .094$, same as for current operations.

(c) Can you think of a reason why it might still have been optimal for the company to finance the project entirely with new equity capital despite the answers in **(a)** and **(b)**?

The project might offer more future options to the company than does its current operations (the assumption only said that risk won't change). With a higher value of future options, the optimal capital structure will be further to the left, i.e. debt to net assets should be smaller. That is what **(a)** achieves.

4. Give at least two completely different sets of assumptions, either one of which is sufficient to conclude that the market portfolio is an efficient portfolio?

One set is the set of assumptions that we used in class to derive CAPM, in which the market portfolio lies on the optimal (efficient) capital market line:

(a) Every investor makes choices based on the effect on her entire portfolio, not based on the individual investment characteristics alone.

(b) Every investor will choose her whole portfolio to have higher μ if σs are the same.

(c) Every investor will choose her whole portfolio to have lower σ if μs are the same.

(d) There exists an investment with $\sigma = 0$ (i.e. a risk-free investment)

(e) All assets trade freely, long or short, at any time, in any amount large or small.

(f) There exists at least one investor whose entire portfolio contains a pro-rata share of all possible investments in the market (i.e. a representative agent).

Another set can be found in the answer to problem #6.6, so refer to the solution manual.

1. (a) The market trades freely, immediately, and with full information for all investors

   (b) Investors are rational, risk-averse maximizers of expected end of period utility

   (c) Investors have homogeneous expectations

   (d) All assets are perfectly divisible and marktable