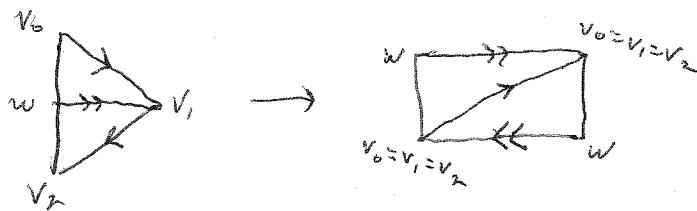
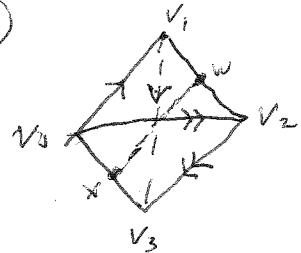


after 131

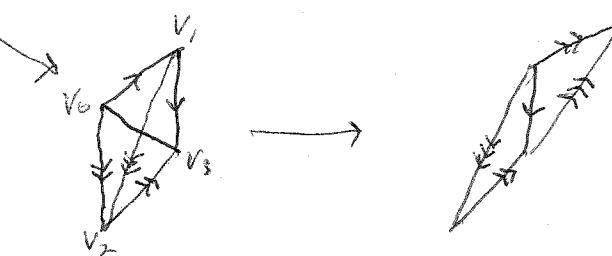
① \mathbb{H}^1 a Möbius Strip



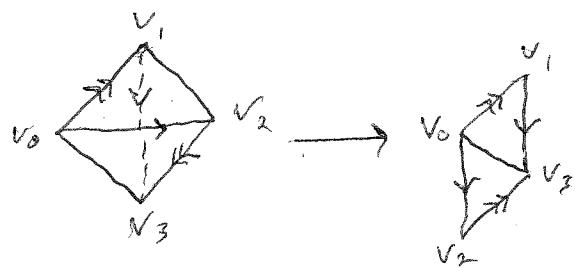
②



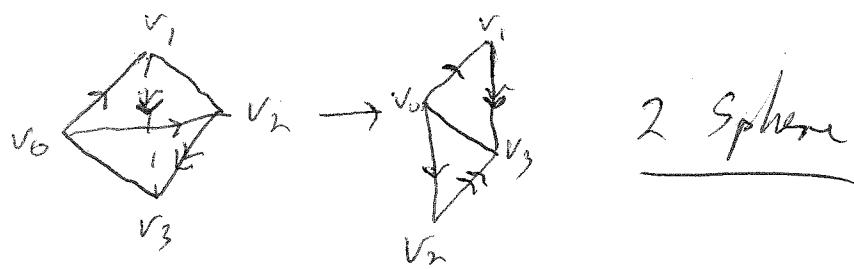
- Retract by projecting parallel to the line from w to x onto $[v_0, v_1, v_3] \cup [v_0, v_2, v_3]$
- There is an obvious homotopy making it a deformation retract



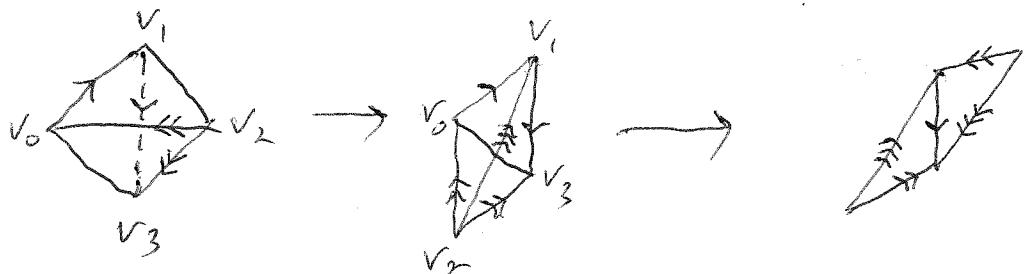
Klein Bottle



Torus



2 Sphere



\mathbb{RP}^2

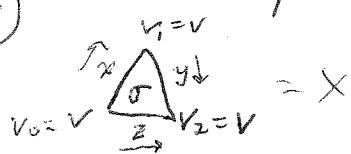
(3) Let v_0, v_1, \dots, v_n be the basis unit vectors of \mathbb{R}^{n+1} .

For any $\{\pm t_i\}$ s.t. $0 \leq t_i \leq 1$ & $\sum t_i = 1$ project radially from $\sum \pm t_i v_i$ to the unit sphere $S^n \subset \mathbb{R}^{n+1}$.

Then each n -simplex $[\pm v_0, \pm v_1, \dots, \pm v_n]$ projects to an simplex in S^n .

Identifying antipodal points in S^n gives a collection of

(4) n -simplices $[v_0, \pm v_1, \dots, \pm v_n]$ in \mathbb{RP}^n



$$0 \rightarrow C_2 \xrightarrow{\partial_2} C_1 \xrightarrow{\partial_1} C_0 \xrightarrow{\partial_0} 0.$$

$$0 \longrightarrow \mathbb{Z} \xrightarrow{\quad} \mathbb{Z} \oplus \mathbb{Z} \oplus \mathbb{Z} \xrightarrow{\quad} \mathbb{Z} \xrightarrow{\quad} 0$$

↑ x y z ↓
 v

$$\mathbb{Z} \xrightarrow{\quad} \mathbb{Z} + \mathbb{Z} - \mathbb{Z}$$

$$x \longmapsto v - v = 0$$

$$y \longmapsto v - v = 0$$

$$z \longmapsto v - v = 0$$

$$\mathbb{Z} \oplus \mathbb{Z} \oplus \mathbb{Z}$$

$$\begin{matrix} / & | & | \\ x & y & \end{matrix}$$

$$\ker \partial_2 = 0$$

$$\underline{H_2(X) \cong 0}$$

$$\ker \partial_1 = \mathbb{Z} \oplus \mathbb{Z} \oplus \mathbb{Z}$$

$$\text{im } \partial_2 = \mathbb{Z}$$

$$\ker \partial_0 = \mathbb{Z}$$

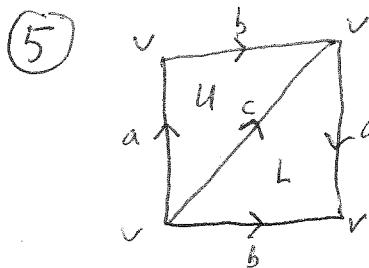
$$\text{im } \partial_1 = 0$$

$$H_1(X) \cong \mathbb{Z} \oplus \mathbb{Z}$$

$$\underline{[x][y]}$$

$$H_0(X) \cong \mathbb{Z}$$

$$\underline{[]}$$



$$0 \rightarrow \mathbb{Z} \oplus \mathbb{Z} \xrightarrow{\quad} \mathbb{Z} \oplus \mathbb{Z} \oplus \mathbb{Z} \xrightarrow{\quad} \mathbb{Z} \rightarrow 0$$

u L a b c v

$$\begin{aligned} uL &\rightarrow a+b-c & aL &\rightarrow v-v=0 \\ L &\rightarrow a-b+c & bL &\rightarrow v-v=0 \\ u+L &\rightarrow 2a & cL &\rightarrow v-v=0 \\ u-L &\rightarrow 2(a-c) \end{aligned}$$

$$\ker \partial_1 = \mathbb{Z} \oplus \mathbb{Z} \oplus \mathbb{Z}$$

$$a+b-c \quad b+c \quad c$$

$$ker \partial_2 = 0$$

100

$$H_2(K) \approx 0$$

$$k\pi d_0 = \pi$$

$$\lim \mathcal{J}_1 = 0$$

$$H_1(K) \cong \mathbb{Z}_2 \oplus \mathbb{Z}_2$$

$$H_0(k) \propto \bar{\epsilon}$$

⑥ With the given identifications the Δ -complex has one vertex v , one edge a_i for each $0 \leq i \leq n$ and one face U_i for each $0 \leq i \leq n$.
 $\partial_1 a_i = v - v = 0$ for all i . $\partial_2 U_0 = a_0 + a_n - a_0 = a_n$ (otherwise U_0 won't be ~~edge~~ part of a Δ -complex). For $i \geq 1$, $\partial_2 U_i = 2a_i - a_{i-1}$.
So $\ker \partial_2 = 0$ and $H_2 \cong 0$.

So $\ker \partial_2 = 0$ and $H_2 \approx 0$

$$\ker \partial_1 = \mathbb{Z} \oplus \mathbb{Z} \oplus \dots \oplus \mathbb{Z}$$

$$\text{Ans} = \frac{a_0}{2a_1 - a_0} \frac{a_1}{2a_2 - a_1} \cdots \frac{a_n}{2a_{n+1} - a_n}$$

$$H_2 \approx \frac{2\pi}{2a_{10}} \frac{2\pi}{2a_{20}} \frac{2\pi}{2a_{n0}}$$

→ because $a_0 = -(2a_1 - a_0) - 2(2a_2 - a_1) - \dots - \frac{1}{2}(2a_n - a_{n-1}) + 2^na_n$

(7)

$$\text{Identifying } [v_0, v_1, v_2] = [v_3, v_1, v_2] \in S$$

$$[v_0, v_1, v_3] = [v_0, v_2, v_3] = F$$

$$[V_0, V_1] = [V_2, V_4] = a, \quad [V_1, V_3] = [V_2, V_5] = b, \quad [V_0, V_3] = c, \quad [V_0, V_5] = d$$

$$\partial_3(\Delta^3) = S - F + F - S \stackrel{?}{=} 0$$

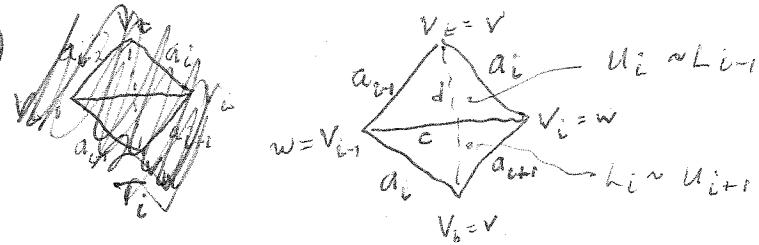
$$\text{so } \ker \partial_3 = \bigoplus_{n=1}^{\infty} \mathbb{Z} \text{ and } H_3 \cong \mathbb{Z}$$

$\partial_2(F) = b - c + a = c$ so $\ker \partial_2 = 0$ and $\text{Im } \partial_2 \neq 0$.

$$\mathcal{D}_1(a) = V_1 - V_D, \mathcal{D}_1(b) = V_B - V_1, \mathcal{D}_1(c) = 0, \mathcal{D}_1(D) = 0$$

$$\phi_1(\gamma) = V_1 - V_D, \quad \phi_2(\gamma) = V_2 - V_1, \quad D_1(C) \geq D, \quad D_1(D) \geq D, \quad \text{ker } D_2 = \mathbb{Z} \oplus \mathbb{Z} \quad \text{im } D_2 = \mathbb{Z} \frac{N_0 \otimes \mathbb{Z}}{\mathbb{Z}}$$

⑧

 T_i

$$n \Delta^3 \text{ simplices } T_i = [v_t, v_b, v_{i-1}, v_i]$$

$$2n \Delta^2 \text{ simplices } L_i = [v_b, v_{i-1}, v_i] \sim [v_t, v_i, v_{i+1}] = U_{i+1}$$

$$R_i = [v_t, v_b, v_i]$$

$$n+2 \Delta^1 \text{ simplices } a_i = [v_b, v_i] \sim [v_b, v_{i-1}]$$

$$c = [v_i, v_{i+1}] \sim [v_{i-1}, v_i]$$

$$d = [v_t, v_b]$$

$$2 \Delta^0 \text{ simplices } v_t = v_b = v$$

$$v_i = \dots = v_n = w$$

$$\partial_3(T_i) = L_i - L_{i-1} + R_i - R_{i-1}, \quad \partial_3(\sum T_i) = 0 \quad \text{and this is the only relation in } \ker \partial_3$$

$$\therefore \ker \partial_3 = \mathbb{Z} \quad H_3(X) \approx \mathbb{Z}$$

$$\partial_2(L_i) = c - a_{i+1} + a_i, \quad \partial_2(R_i) = a_{i+1} - a_i + d, \quad \partial_2(L_i + R_i) = c + d$$

$$\partial_2(\sum L_i) = n c, \quad \partial_2(\sum R_i) = n d \quad \text{and these are the only relations in } \ker \partial_2$$

$$\therefore \ker \partial_2 = 0 \quad H_2(X) \approx 0$$

$$\partial_1(a_i) = v_i - v_t = w - v, \quad \partial_1(c) = v_{i+1} - v_i = w - w = 0, \quad \partial_1(d) = v_b - v_t = v - v = 0$$

$$\therefore \partial_1(a_i - a_j) = 0 \quad \text{and } \ker \partial_1 = \{\text{generated by } a_i - a_j, c, d\}$$

~~the~~ $\ker \partial_2 = \{\text{generated by } c - a_{i+1} + a_i, c + d, c\}$

$\ker \partial_2 = \{\text{generated by } c - a_{i+1} + a_i, c + d, nc\}$

$$H_1(X) \approx \mathbb{Z}/n\mathbb{Z} \approx \mathbb{Z}_n$$

$$\ker \partial_0 = \mathbb{Z} \oplus \mathbb{Z} \oplus \mathbb{Z} \oplus \mathbb{Z}$$

$$v \quad w \quad w-v \quad v$$

$$\ker \partial_1 = \mathbb{Z}_{w-v} \quad \text{so} \quad H_0(X) \approx \mathbb{Z}$$

(9) If the simplices are Δ^{2k} and Δ^{2k-1} ,

$$\partial_{2k}(\Delta^{2k}) = \sum_{i=0}^{2k} (-1)^i \Delta^{2k-i} = \Delta^{2k-1}$$

$$\partial_{2k-1}(\Delta^{2k-1}) = \sum_{i=0}^{2k-1} (-1)^i \Delta^{2k-1-i} = 0$$

So for $n=2m$

$$0 \rightarrow C_{2m}(\Delta^{2m}/n) \rightarrow C_{2m-1}(\Delta^{2m}/n) \rightarrow \dots \rightarrow C_1(\Delta^{2m}/n) \rightarrow C_0(\Delta^{2m}/n) \rightarrow 0$$

$$0 \rightarrow \mathbb{Z} \xrightarrow{\text{id}} \mathbb{Z} \xrightarrow{\text{id}} \mathbb{Z} \xrightarrow{\text{id}} \dots \xrightarrow{\text{id}} \mathbb{Z} \xrightarrow{\text{id}} \mathbb{Z} \rightarrow 0$$

$$\frac{H_k(\Delta^{2m}/n) \approx 0 \text{ for } k > 0}{H_0(\Delta^{2m}/n) \approx \mathbb{Z}}$$

and for $n=2m-1$

$$0 \rightarrow C_{2m-1}(\Delta^{2m-1}/n) \rightarrow C_{2m-2}(\Delta^{2m-1}/n) \rightarrow \dots \rightarrow C_1(\Delta^{2m-1}/n) \rightarrow C_0(\Delta^{2m-1}/n) \rightarrow 0$$

$$0 \rightarrow \mathbb{Z} \xrightarrow{\text{id}} \mathbb{Z} \xrightarrow{\text{id}} \mathbb{Z} \xrightarrow{\text{id}} \dots \xrightarrow{\text{id}} \mathbb{Z} \xrightarrow{\text{id}} \mathbb{Z} \rightarrow 0$$

$$\frac{H_{2m-1}(\Delta^{2m-1}/n) \approx \mathbb{Z}}{H_0(\Delta^{2m-1}/n) \approx 0}$$

$$\frac{H_k(\Delta^{2m-1}/n) \approx 0 \text{ for } 2m-1 > k > 0}{H_0(\Delta^{2m-1}/n) \approx \mathbb{Z}}$$