Math 369
Spring 2008
Final Examination Solutions
May 2, 2008

This is an open-book take-home exam. You may work with textbooks and notes but do not consult any other person. Show all of your work and put your name on all papers. The exam is due back by 5 PM on Thursday May 8. You may place it in my box in the faculty mailroom or under my office door.

1. The value today of a stock is \$10 per share. The risk free rate is 3% per year. The volatility of the stock is 30% on an annualized basis. Using a binomial tree with 2 steps per year, how much larger is the value of an American put on the stock than the value of a European put on the stock if they both expire in 3 years and have a strike price of \$3 per share?

Solution: $p = 1 - p = \frac{1}{2}$; $u = e^{\frac{1}{2}(.03) - \frac{1}{4}(.3)^2 + \sqrt{\frac{1}{2}}(.3)} = 1.2271$; $d = e^{\frac{1}{2}(.03) - \frac{1}{4}(.3)^2 - \sqrt{\frac{1}{2}}(.3)} = .80281$. $d^5 = 3.335$, so no values on the tree prior to time 2 are below the strike price so there will be no early exercise. Thus American put value equals European put value in this case.

2. The Black-Scholes formula for the price of a call option is

$$c = S\Phi(d_1) - Xe^{-rT}\Phi(d_2)$$

where d_1 and d_2 are certain expressions that you can evaluate. Once you know d_1, the value $\Phi(d_1)$ is obtained from a table of normal probability values (or a computer function of those same normal probability values). Presumably, then, $\Phi(d_1)$ is the probability of some event. Explain what event that is.

Solution: $\Phi(d_1)$ is not the probability of any event. It is the expected value of $e^{-rT}\frac{\bar{S}(T)}{S}$ where $\bar{S}(T) = S(T)$ for $S(T) \geq X$ and $\bar{S}(T) = 0$ for $S(T) < X$. It is just an accident of the mathematical form of the lognormal density function that this complicated expected value can be found in a table of probability values. (note: using concepts not covered in class, it is possible to identify $\Phi(d_1)$ as the probability that $S(T) \geq X$ under the probability measure that corresponds to using $S(t)$ rather than the risk-free bank account as numeraire.)

3. You want to build a portfolio consisting only of shares of two stocks A and B. They have expected returns of 5% and 12%, respectively, and their standard deviations of return are 2% and 8%, respectively. The correlation coefficient between their returns is 0.5. The risk free rate is 3%. What is the optimal portfolio constructed from just those two stocks? What is optimal portfolio constructed from just those two stocks, plus risk-free borrowing or investing, and having a 3% standard deviation of return ?

Solution:

$$\begin{aligned}
w_1 &= \frac{(.05-.03)(.08)^2-(.12-.03)(.5)(.02)(.08)}{(.12-.03)(.02)^2+(.05-.03)(.08)^2-(.05+.12-2(.03))(.5)(.02)(.08)} \\
&= .7368 \\
w_2 &= 1-.7368=.2632 \\
\sigma_P &= \left[(.7368)^2(.02)^2+(.2632)^2(.08)^2+2(.5)(.7368)(.2632)(.02)(.08)\right]^{\frac{1}{2}} \\
&= .03116 \\
\sigma &= .03 \text{ for } \frac{.03}{.03116}\tilde{r}_P+(1-\frac{.03}{.03116})\tilde{r}_f \\
\bar{w}_1 &= \frac{.03}{.03116}.7368=.7094 \\
\bar{w}_2 &= \frac{.03}{.03116}.2632=.2534 \\
\bar{w}_f &= (1-\frac{.03}{.03116})=.0372
\end{aligned}$$

4. You plan to use 20,000 blank silicon wafers as raw material for your chip manufacturing business in each of the next 20 years. The current market price for blank silicon wafers is \$5 per wafer. Your leading technical experts tell you that the price of silicon wafers will increase by 10% per year for the next five years, by 5% for each of the following five years, and by 3% per year after that. In order to be prepared for your chip needs you have already purchased European call options on 20,000 blank silicon wafers with expiry dates in each of the next 20 years (i.e. 20,000 per year.) To help finance the purchase of the call options, you have sold European put options on 20,000 blank silicon wafers with expiry dates in each of the next 20 years (i.e. 20,000 per year.) The strike price for both the call and the put options is \$6 per wafer. If the risk free rate is 3% what is the value of your net position in the options, calls and puts combined? You can assume that the cost of storing unused wafers from year to year is negligible.

Solution: Since storage costs are negligible you can use put-call parity to value the net position in terms of today's \$5 per wafer market value and the risk free present value of the \$6 strike prices. **Always** prefer an observed market price over any experts' forecasts.

$$\begin{aligned}
20,000\cdot 20\cdot \$5-20,000\cdot(\text{20 year 3\% annuity PV})\cdot\$6 &= \\
&= \$2,000,000-41,785,297 \\
&= \$214,703
\end{aligned}$$

5. Consider the situation of exercise 5.14 in the text. If the expected returns on each balance sheet category are as follows:

short term asets 2%
U.S. Treasury bonds 6%
loans 10%
sort term liabilities 3%
deposits 2%

what is the Sharpe ratio of the equity holders' position before and after taking the recommended T-Bond futures position as a hedge? If you need to make an assumption specify clearly what you are assuming.

Solution: To get the Sharpe ratio you need to assume a risk-free rate. I assumed .02 considering the given returns on short term assets and liabilities. Many of you assumed .06 based on the given long term government bond yields. As long as you specified what you were assuming you got full credit. Using my assumption, and taking the standard deviation from the solution manual, before the hedge:

$$\begin{aligned}\text{Sharpe ratio} &= \frac{\frac{100}{100}.02 + \frac{200}{100}.06 + \frac{700}{100}.10 - \frac{50}{100}.03 - \frac{850}{100}.02 - .02}{.574482} \\ &= 1.1053\end{aligned}$$

After the hedge, you need to be careful. The theoretical logic of a hedge using futures is that you change the standard deviation without laying out any funds up front and without changing the expected value at all. In particular, the theoretical expected return on the hedge is zero. (i.e. positive in unfavorable scenarios and negative in favorable scenarios, average zero if you have hedged perfectly). You are not investing in treasury bonds when you hedge with treasury bond futures. Your expected return is not the treasury bond return. Again, using the standard deviation from the solution manual, after the hedge:

$$\begin{aligned}\text{Sharpe ratio} &= \frac{\frac{100}{100}.02 + \frac{200}{100}.06 + \frac{700}{100}.10 - \frac{50}{100}.03 - \frac{850}{100}.02 - .02}{.503592} \\ &= 1.2609\end{aligned}$$

In practice, it would be wise to see how much of an unfavorable drift (actual expected return on the hedge different from the theoretical zero) you can withstand without reducing the original Sharpe ratio of 1.1053. Using the solution manual's nominal base of 301 million for the hedge, an unfavorable drift of .026 can be seen to give back the original Sharpe ratio:

$$\begin{aligned}\text{Sharpe ratio} &= \frac{\frac{100}{100}.02 + \frac{200}{100}.06 + \frac{700}{100}.10 - \frac{50}{100}.03 - \frac{850}{100}.02 - \frac{301}{100}.026 - .02}{.503592} \\ &= 1.1053\end{aligned}$$

This can arise from the costs of managing the hedge, such as interest on margin requirements, dealer's spread, etc. These would be unlikely to be as large as 2.6%, so the strategy is viable from a cost perspective. Since no hedge is perfect, the 2.6% also gives you an idea of how much margin for error you can live with in the hedging structure and not have given up the benefit of hedging from a risk-reward perspective.

6. Your boss tells you that the market has an expected return of 16% with 20% standard deviation. The risk free rate is 3%. (a) What do you think about that? Then she tells you to keep your thoughts to yourself and use her assumptions. (b) Would you recommend buying a stock that has an expected return of 6% and a covariance of 1% with the market? (c) You buy the stock and the actual return is negative 1%. Give three distinct explanations for how that might have happened, other than your being stupid or your boss being wrong.

Solution: (a) 13% is an extremely aggressive assumption for the market risk premium compared to its history of 9% and current opinion that something less is now to be expected, maybe as low as 5% to 7%.

(b) But keeping the assumption of .13 CAPM says that the required return is $.03 + \frac{.01}{.04}.13 = .0625$, so the forecast return of .06 is insufficient and should be rejected. Note that a more reasonable market risk premium assumption would have led to the conclusion that the return was more than adequate. Also, it would be acceptable to say that .0625 and .06 are not significantly different for purposes of equity analysis and therefore the investment is accepted (that would be huge difference in fixed income investing, but a rounding error in equity investing, where the noise terms are so much larger.)

(c) Some reasons that could be given include

- The expected return was reasonable, the difference from actual was not statistically significant in one period. For example, if the beta of $\frac{.01}{.04}$ consists of $.71\frac{.07}{.20}$ then the 7% unfavorable swing in one period was only a one standard deviation fluctuation.
- Maybe the market as a whole was down in that period. With an alpha of .0225, for example, a 13% market correction coupled with the beta of $\frac{.01}{.04}$ would produce a 1% loss.
- Maybe something changed in the operations or markets of the company to make past market correlations no longer relevant.

7. Over the past 60 months stock A had an average monthly total return of 1.1%, a standard deviation of monthly total return of 5.8%, and a correlation coefficient 0.5 of its monthly total return with the monthly total return of the market. In the same period, stock B had an average monthly total return of 1.0%, a standard deviation of monthly total return of 5.8%, and a correlation coefficient 0.6 of its monthly total return with the monthly total return of the market. The monthly total return of the market over the same period averaged 0.5 % with a standard deviation of 7.1%. This month the market had a total return (a loss) of (3)%, stock A had a total return of 0% and stock B had a total return (a loss) of (1)%. (a) What was the abnormal return this month for stock A and stock B? (b) Does this result indicate superior operating performance by the managers of one company or the other? Give at least two different reasons for your answer.

Solution:

$$
\begin{aligned}
\alpha_A &= .011 \qquad \alpha_B = .01 \qquad \alpha_m = .005 \\
\sigma_A &= .058 \qquad \sigma_B = .058 \qquad \sigma_m = .071 \\
\beta_A &= .40845 \qquad \beta_B = 49014 \\
ABNORM_A &= 0 - [.011 + .40845(-.03)] = .00125 \\
ABNORM_B &= -.01 - [.01 + .49014(-.03)] = -.00523
\end{aligned}
$$

You can draw no conclusions about management performance

- this is only one monthly observation
- the difference between the abnormal performances of A and B is not statistically significant, less than 15% of a monthly standard deviation
- something may have changed in operations or in markets to make historical correlations no longer relevant.

8. Warren Buffet has earned a 24% compound annual return on his investment portfolios over the past 40 years. The compound annual return on the S&P 500 over those same 40 years was 13%. Does this contradict the semi-strong form of the efficient market theory? Give at least four separate and distinct reasons for your answer.

Solution: No. Reasons include:

- He could be a statistical outlier
- He might operate with substantial non-public information
- He may run more risk (have a higher beta) than the market
- His investment universe may be substantially different from the S&P 500 (foreign stocks, private equity, IPOs, etc)
- He may concentrate his trading on times when the market is out of equilibrium

- His own investments might influence the investments of other enough to move the market

9. A stock has a dividend yield of 2% and the company pays 7.5% interest on its long term debt. The ROE based on beginning of year equity is 16%. The are 10 million shares outstanding. The market to book ratio is 1.25 and the share price is $40. The interest payments on long term debt total $2.50 per share. What is the maximum possible growth rate for this company without any new external financing of any kind?

Solution: We are looking for the internal growth rate:

$$\begin{aligned}
g &= \frac{PB \cdot NI}{BV + D} \text{ with beginning of year } BV \\
&= PB \cdot ROE \cdot \frac{BV}{BV + D} \\
&= \frac{NI - DIV}{NI} \cdot ROE \cdot \frac{1}{1 + \frac{D}{BV}} \\
&= \left(ROE - \frac{DIV}{BV} \right) \cdot \frac{1}{1 + \frac{\frac{Int}{BV}}{\frac{Int}{D}}} \\
&= \left(ROE - \frac{d \cdot MV}{BV} \right) \cdot \frac{1}{! + \frac{\frac{Int}{MV}\frac{MV}{BV}}{\frac{Int}{D}}} \\
&= (.16 - .02 \cdot 1.25) \cdot \frac{1}{1 + \frac{\frac{2.50}{40} 1.25}{.075}} \\
&= .0661
\end{aligned}$$

10. For years Vega Motors has plowed back 60% of earnings while making 20% return on equity and maintaining a 2% dividend yield. They have been able to keep their debt ratio unchanged. The market priced Vega's shares as if the growth rates corresponding to this financial performance could continue forever. By what % and in what direction will Vega's share price change if the company suddenly announces, in a complete surprise to the market, that it has no further opportunities for profitable growth beyond its current scale of operations, now plans no further growth at all, and will begin to pay out all of its earnings as dividends each year?

Solution: All of the current $PVGO$, present value of growth opportunities, will disappear from the value.

$$
\begin{aligned}
\frac{PVGO}{P} &= \frac{1}{P} \cdot \left(P - \frac{eps}{r} \right) \\
&= \frac{1}{P} \cdot \left(P - \frac{\frac{DIV}{1-PB}}{d+g} \right) \\
&= \frac{1}{P} \cdot \left(P - \frac{DIV}{(1-PB)\,(d + PB \cdot ROE)} \right) \\
&= 1 - \frac{d}{(1-PB)\,(d + PB \cdot ROE)} \\
&= 1 - \frac{.02}{(1-.60)(.02 + .60 \cdot .20)} \\
&= .6429
\end{aligned}
$$

11. Assume that you believe the basic premises of the Pecking Order Theory for capital structure. Despite that belief, explain why it still might make sense for a company to take on (borrow) new long term debt to finance a project even though it has enough cash and marketable securities easily to finance the project without borrowing. Use at least one formula or diagram to illustrate or support your reasoning. Be specific about any assumptions you make.

Solution: Remember that you were told to assume that you believe the Pecking Order Theory. So you believe that borrowing when you could have used internal funds sends a negative signal to the market, they believe you are hiding something, and they reduce your value accordingly. To go ahead and borrow anyway, you must believe that something about the borrowing NOT ONLY adds value but also add MORE value than the value lost through the negative signal. So your answer must point out (a) that the borrowing creates valuable tax shields that exceed the erosion of value due to loss of flexibility and to increase in expected value of costs of financial distress and (b) that the net remaining value created is still larger than the loss of value due to the negative signal.

12. Gimmel Inc. has a beta of 0.5 on its equity, 40% debt in its capital structure, with the debt being valued by the market as essentially risk-free at a 6% pre-tax annual yield. The expected return on the entire market is 18%. Gimmel is considering a project called Gamma to develop a chain of high-end urban retail outlets for its products that it expects will yield 25% annually on an after-tax basis. The main competitor will be Himmel Inc. which ought to have about the same risk characteristics as Gamma. Himmel's equity beta is 2.2 and it has 10% debt in its capital structure. Assume that the marginal tax rate for both companies is 50% and that the Gamma project will be funded with 40% debt, 60% retained earnings. From a purely financial point of view, should

Gimmel proceed with the Gamma project? Give specific financial analyses and reasons.

Solution: In Himmel

$$
\begin{aligned}
r_E &= .06 + 2.2(.18 - .06) \\
&= .324 \\
WACC &= .9(.324) + .1 \cdot .5 \cdot .06 \\
&= .2946 \\
&= \rho(1 - .5 \cdot .1) \text{ so} \\
\rho &= .3101
\end{aligned}
$$

For Gamma with 40% debt

$$
\begin{aligned}
WACC &= .3101(1 - .5 \cdot .4) \\
& \quad .2481
\end{aligned}
$$

Since Gamma return of 25% exceeds the required 24.81%, proceed with the project.