

The Effect of Global Warming On Financial Discounting Methodology

James G. Bridgeman, FSA

University of Connecticut

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Actuarial Framework

PRESENT VALUE - what to set aside today to match a future cash need.

- $PV(0, C(t)) = C(t)e^{-rt}$

$C(t)$ a random variable; r the discount rate (for risk and time)

- $PV(0, C(t)) = C(t)e^{-r[C(t)]t} = C(t)e^{-\int_0^t r[C(s)]ds}$

$r[C(t)]$ a real functional reflecting risk in $C(t)$ and/or in the bond markets

- $\int_0^\infty PV(0, C(t))dt = \int_0^\infty C(t)e^{-\int_0^t r[C(s)]ds} dt$

$r[C](s)$ a functional reflecting risk in the stochastic process $C(t)$ and/or bond markets and taking values in a space of stochastic processes

- $\mathbb{E} \left[\int_0^\infty PV(0, C(t))dt \right] = \mathbb{E} \left[\int_0^\infty C(t)e^{-\int_0^t r[C](s)ds} dt \right]$ gives the
expected value of the random variable for the present value

DISCOUNT RATES

- From $PV(0, C(t)) = C(t)e^{-\int_0^t r[C](s)ds}$ notice that

$$r[C](s) = -\frac{d}{dt} \ln \left\{ e^{-\int_0^t r[C](s)ds} \right\}_{t=s}$$

- In fact, if we have $PV(0, C(t)) = C(t)P[C](t)$ for any present value functional $P[C](t)$ taking values in a space of stochastic processes of bounded variation we can find the implicit discount rate functional:
$$r[C](s) = -\frac{d}{dt} \ln \{P[C](t)\}_{t=s}$$

Stern Review: The Economics of Climate Change

- October 30, 2006
- Sir Nicholas Stern, head of UK government economics service
- Headlines: If nothing is done to arrest it, the expected value of the present value of the future financial value of the effects of global warming could be equivalent to a 20% decline in world real GDP per capita, starting now and lasting forever. Such a decline in GDP would be a catastrophe equivalent to all the wars and great depressions of the twentieth century combined.
- Fine print: The study actually concluded that the expected value is in a range of 5% to 20% decline in world real GDP per capita, but the 20% possibility should be taken seriously.
- Prescription: We can avoid it if we start immediately to sacrifice 1% of world real GDP per capita annually to arrest warming. A no-brainer. (Fine print: mainstream studies range from 0% to 5%).

Yikes! What Are the Numbers?

EXPECTED VALUE OF REDUCTION IN WORLD REAL GDP PER CAPITA CAUSED BY GLOBAL WARMING

Model	2060	2100	2200&BEYOND
Typical*			2.2%
add Catastrophes*	0.2%	0.9%	5.3%
add Feedbacks*			7.3%
add non-Market Effects*		2.9%	13.8%
add new Sensitivity Est.**	1.3%	5.9%	24.4%

source: *buried in the text on pages 155 and 156

**buried in the text on page 156, science not solid yet

How Certain Are The Numbers?

RANGE OF VALUE OF REDUCTION IN WORLD REAL GDP PER CAPITA CAUSED BY GLOBAL WARMING IN 2200 & BEYOND

Model	5%-ile	Mean	95%-ile
Typical Models*		2.2%	
add Catastrophes*	0.6%	5.3%	13.4%
add Feedbacks*	0.9%	7.3%	17.9%
add non-Market Effects*	2.9%	13.8%	35.2%
add new Sensitivity Est.**		24.4%	

sources: *buried in the text on pages 155 and 156

**buried in the text on page 156, science not solid yet

What Are The Present Values?

Balanced Growth Equivalent (BGE): The reduction in world real GDP per capita applied now and in all future years that would produce the same present value

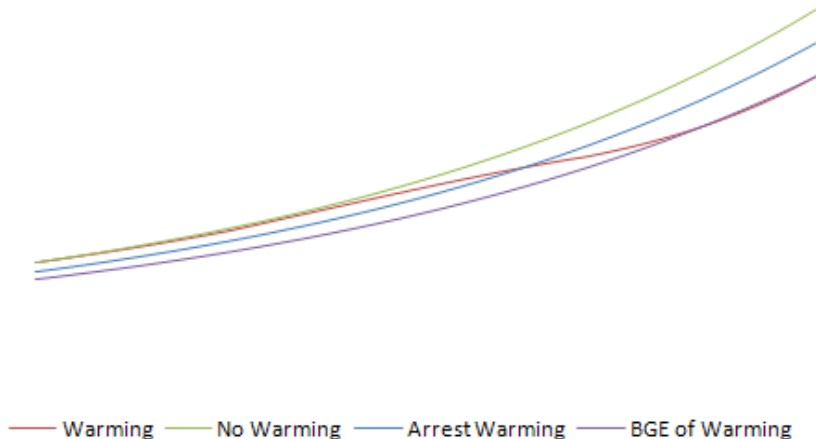
RANGE OF BGE OF REDUCTION IN ALL FUTURE WORLD REAL GDP PER CAPITA CAUSED BY GLOBAL WARMING

Model	5%-ile	Mean	95%-ile
Typical Models	0.3%	2.1%	5.9%
add Catastrophes	0.6%	5.0%	12.3%
add Feedbacks	0.9%	6.9%	16.5%
add non-Market Effects	2.7%	14.4%	32.6%
add new Sensitivity Est.		not given	
recommended upper bound*		20%	

sources: chart on page 163

*judgmental, text on page 164

World GDP Over Centuries



What Kind of Present Values Are Those?

RANGE OF VALUE OF REDUCTION IN WORLD REAL GDP PER CAPITA CAUSED BY GLOBAL WARMING IN 2200 & BEYOND

Model	5%-ile	Mean	95%-ile
Typical Models		2.2%	
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Partial Explanation (from commentators)

- They discounted at only $\delta = 0.1\%$ per year
 - Then why aren't the effects proportional?
 - And how can a BGE be higher than every one of the future values?
- They used an inappropriate elasticity of marginal utility $\eta = 1$
 - This has the effect of valuing equal percentage changes in wealth equally whether for paupers or millionaires
 - 1.3% baseline annual growth in world real GDP per capita makes a lot more future millionaires and a lot fewer future paupers, so they are valuing a reduction in future caviar supplies equally with a reduction in current bread supplies

Stern Review Explanation

- Why discount at only $\delta = 0.1\%$ per year?
 - We have no *ethical* right to value our welfare above that of our great-great-great-great-great grandchildren just because we are alive and they are not. The *ethical* pure discount rate is 0%. But there is some chance that we won't have any great-great-great-great-great grandchildren at all (asteroids, epidemics, etc. might make the human race extinct) so we can discount for that probability. Make it 0.1% per year, even though that's probably too big
- What does the elasticity of marginal utility (whatever that is) have to do with present values?
 - Financial discounting works by examining the marginal effect on the welfare of the world from a small change in circumstances. If the circumstances you are evaluating change the world entirely, then you cannot use a technique grounded in marginal effects. Instead sum up all future welfare effects (discounted only for the probability of human extinction) of the circumstances you are evaluating.

- But why use that $\eta = 1$ value in calculating future welfare?
 - Not much of an answer was given. Essentially, they claimed that the empirical economics literature doesn't clearly support any other value as being implied by current interest rates. They didn't even mention an *ethical* dimension to this question (valuing future caviar shortages the same as bread shortages today).

Financial Discounting Is Only For Marginal Effects?

PV of Welfare = *utility of consumption* \times *pure time preference*

$$W(0) = \int_0^{\infty} U(C(t)) e^{-\delta t} dt$$

$$\Delta W(0) = \int_0^{\infty} \Delta C(t) \frac{dU}{dC}(C(t)) e^{-\delta t} dt$$

$$\text{Discount rate at } s \text{ is} = -\frac{d}{ds} \ln \left[\frac{dU}{dC}(C(s)) \right] + \delta$$

$$\text{Let } \frac{dU}{dC}(C) = C^{-\eta} \text{ for some } \eta \geq 1$$

$$\text{i.e. } U(c) = K + \frac{1}{1-\eta} C^{1-\eta} \text{ or } = K + \ln C \text{ for } \eta = 1$$

$$\text{Discount rate at } s \text{ is} = \eta \frac{\frac{dC}{ds}(s)}{C(s)} + \delta \text{ for marginal } \Delta C(t), t \geq s$$

$$\eta = \text{the elasticity of marginal utility}$$

So Financial Discounting Depends On The Economic Path

Discount rate at s is $= \eta \frac{\frac{dC}{ds}(s)}{C(s)} + \delta$ for marginal $\Delta C(t)$, $t \geq s$

- In a faster growing economy, you discount at a higher rate
- In a negatively growing economy ($\frac{dC}{ds}(s) < 0$), you might even discount at a negative rate! (if $-\eta \frac{\frac{dC}{ds}(s)}{C(s)} \geq \delta$)
- This is not unlikely if δ has been chosen quite small!
- But what if you are discounting a disturbance to the entire economic path?
 - $C(s) \longrightarrow C'(s)$ for all s
 - Do I use $\frac{dC}{ds}(s)$ or $\frac{dC'}{ds}(s)$ to determine my discount rate?
- Stern Review says you give up discounting financial values and instead go back to $W(0) = \int_0^{\infty} U(C(t))e^{-\delta t} dt$

Stern Review: You Can't Discount Financial Values

- If you can't discount how do you get a present value?
- Well, they never actually calculate present values of financial variables, only of welfare.
- If $W(0) = \int_0^{\infty} U(C(t))e^{-\delta t} dt = \int_0^{\infty} U(C'(t))e^{-\delta t} dt = W'(0)$ then say that $C(t)$ and $C'(t)$ have "the same present value"
- $C(t)$ might represent the BGE path at a lower growth rate than a world without global warming; $C'(t)$ might represent the path of the world with global warming
- Notice that the BGE path won't have any negative value of $\frac{dC}{ds}(s)$; a Monte Carlo generated global warming path has a high likelihood of $\frac{dC'}{ds}(s) < 0$ for long stretches of time.
- All the present value assertions in the Stern Review come about in this way.
- Why does that matter? We're not discounting financial values anymore, we're only discounting welfare.

But We Are Discounting Financial Values!

- Let $C(t)$ be the BGE path at a lower growth rate than the world without global warming and $C'(t)$ be a Monte Carlo generated global warming path.
- At each t let $\Delta C(t) = C(t) - C'(t)$ and define $C(t, p)$ for $0 \leq p \leq 1$ by $\frac{\partial C(t, p)}{\partial p} = \Delta C(t)$. Use $C(t, p)$ to define
$$W(0, p) = \int_0^\infty U(C(t, p))e^{-\delta t} dt.$$

$$\text{Then } \Delta W(0) = \int_0^1 \frac{\partial W}{\partial p}(0, p) dp$$

$$\begin{aligned} \text{So } \Delta W(0) &= \int_0^1 \int_0^\infty \frac{\partial C(t, p)}{\partial p} \frac{dU}{dC}(C(t, p)) e^{-\delta t} dt dp \\ &= \int_0^\infty \Delta C(t) \int_0^1 e^{-\int_0^t \left(\eta \frac{\frac{\partial C}{\partial s}(s, p)}{C(s, p)} + \delta \right) ds} dp dt \end{aligned}$$

The Implicit Discount Rate In Stern

- This implies

$$\begin{aligned}\text{discount rate} &= -\frac{d}{dt} \ln \int_0^1 e^{-\int_0^t \left(\eta \frac{\frac{\partial C}{\partial s}(s,p)}{C(s,p)} + \delta \right) ds} dp \\ &= \frac{\int_0^1 \left(\eta \frac{\frac{\partial C}{\partial s}(t,p)}{C(t,p)} + \delta \right) e^{-\int_0^t \left(\eta \frac{\frac{\partial C}{\partial s}(s,p)}{C(s,p)} + \delta \right) ds} dp}{\int_0^1 e^{-\int_0^t \left(\eta \frac{\frac{\partial C}{\partial s}(s,p)}{C(s,p)} + \delta \right) ds} dp}\end{aligned}$$

- Which is a weighted average of the discount rates across a continuous set of paths connecting the BGE path $C(t)$ with the Monte Carlo global warming path $C'(t)$, each discount rate weighted by the discount factor back to today implied by its own path. .

The Implicit Discount Rate In Stern

- If $\frac{\partial C}{\partial s}(s, p) \leq 0$ for a significant range of p , which will be the case when $\frac{dC'}{ds}(s) \leq 0$, and if δ is small, then this implicit discount rate in Stern's modeling can be negative, especially because the negative values will have higher weight (lower discount rates mean higher discount factors) than the constant positive discount rates at the BGE $C(t)$ end of the range of paths.
- This discounting at a negative discount rate for stretches of time in some of the Monte Carlo runs is especially pernicious because Stern approximates $\int_{\text{year } 2200}^{\infty}$ by a simple growing perpetuity on each Monte Carlo path.
- Well, that's the Stern Review methodology. What about his choices for the values of δ and η ?

Comparitive Values of Parameters - BGE Decline Due To Warming for Typical Models plus Cats, Feedback & Non-Market Effects

η	1.5	1.0	1.5	1.0	yr 2200	1.0
δ	1.5	1.5	0.1	0.0		0.1
5%-ile	0.6%	0.8%	2.0%	3.2%	2.9%	2.7%
mean	2.8%	4.2%	10.2%	16.9%	13.8%	14.4%
95%-ile	6.8%	10.1%	20.0%	38.4%	35.2%	32.6%
judgmental	4%	6%	13%	24%		20%

source	JB	Annex	Annex	JB	p. 156	p.163
conclusion	too low	too low	too low	good!	rats!	well,OK

- Stern couldn't get the answer he wanted with $\eta > 1$ or $\delta > 0$, but then had to dial δ up just a bit in order avoid looking ridiculous.

Unfortunately, at $\delta = 0.1$ there are still some ridiculous data points

Speaking of Ethics

- The baseline growth rate in real per capita GDP assumed in Stern's modeling is 1.3% per annum.
- With world GDP per capita of \$7,600 in 2006 (Stern) that means that without global warming world real per capita GDP in 2200 would be \$93,121.
- A 13.8% decline in 2200 world real GDP per capita caused by global warming would leave it at \$80,270.
- How much should the poverty end of today's \$7,600 average be expected to pay to keep that unfortunate average citizen in 2200 from having to live on only \$80,270 instead of \$93,121?
- By setting $\eta = 1$ most of that decline from \$93,121 to \$80,270 is reflected today. Tomorrow's caviar shortage is equated to today's bread shortages.
- Should we really believe even a low growth rate like 1.3% as lasting forever?

A Few More Thoughts

- Maybe the social consequences of a falling GDP (chaotic and violent unrest, organized warfare, etc.) are the same whether it's falling from a high level to a slightly less high level or from a low level to a slightly lower level?
- In that case, maybe it makes sense to discount future values at a negative interest rate when valuing income that would arrive in the midst of such dire circumstances? (Assuming you've properly discounted for the probability of arrival, given such dire circumstances)
- In that case, maybe it makes sense to discount the value of that future decline at a negative interest rate when deciding what it is worth spending today in order to avert/ameliorate the decline?
- But regardless, these aren't your father's present values! It is a distinct methodological departure. There's a lot to learn from it, but it seems troublesome that the results were summarized as simply "present value" comparisons.

Actuarial Framework

PRESENT VALUE - what to set aside today to match a future cash need.

- $PV(0, C(t)) = C(t)e^{-rt}$

$C(t)$ a random variable; r the discount rate (for risk and time)

- $PV(0, C(t)) = C(t)e^{-r[C(t)]t} = C(t)e^{-\int_0^t r[C(s)]ds}$

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- $\int_0^\infty PV(0, C(t))dt = \int_0^\infty C(t)e^{-\int_0^t r[C(s)]ds} dt$

$r[C](s)$ a functional reflecting risk in the stochastic process $C(t)$ and/or bond markets and taking values in a space of stochastic processes

- $\mathbb{E} \left[\int_0^\infty PV(0, C(t))dt \right] = \mathbb{E} \left[\int_0^\infty C(t)e^{-\int_0^t r[C](s)ds} dt \right]$ gives the
expected value of the random variable for the present value

DISCOUNT RATES

- From $PV(0, C(t)) = C(t)e^{-\int_0^t r[C](s)ds}$ notice that

$$r[C](s) = -\frac{d}{dt} \ln \left\{ e^{-\int_0^t r[C](s)ds} \right\}_{t=s}$$

- In fact, if we have $PV(0, C(t)) = C(t)P[C](t)$ for any present value functional $P[C](t)$ taking values in a space of stochastic processes of bounded variation we can find the implicit discount rate functional:
 $r[C](s) = -\frac{d}{dt} \ln \{P[C](t)\}_{t=s}$
- More generally, the discount rate functional would be a stochastic differential $dR[C](s) = -d \ln P[C](s)$ for a present value functional $P[C](t)$ taking values in a space of semimartingales. Not usual for actuaries, who prefer to restrict the martingales to the $C(t)$ part of their models, not least because this general $dR[C](s)$ could easily be negative at some values s on some paths.

PRESENT VALUE - the value in today's market of a portfolio of securities guaranteed to match a random future cash need.

- $PV(0, C(t)) = \mathbb{E} \left[\tilde{C}[C](t) e^{-\int_0^t \tilde{r}(s) ds} \right]$ which is **not** a random variable.
- $\tilde{r}(s)$ is a stochastic process (called the risk-free rate) reflecting only treasury bond market risk; not dependent upon $C(t)$
- $\tilde{C}[C](t)$ is a functional reflecting all risk in $C(t)$, including its correlations with bond markets, and taking values in a space of stochastic processes
- $\tilde{C}[C](t) = C(t) \frac{d\mu_C}{d\mu}(t)$ where the last factor is a Radon-Nikodym derivative process for a change of measure that accomplishes the preceding concept, and that is parameterized with reference to current market values of the risky securities in the portfolio that is guaranteed to match $C(t)$.

DISCOUNT RATES

- $PV(0, C(t)) = \mathbb{E}_C \left[C(t) e^{-\int_0^t \tilde{r}(s) ds} \right]$ is the more usual expression

with \mathbb{E}_C denoting an expected value with respect to the measure $\mu_C(t)$, given the name "risk-neutral measure"

- But from $PV(0, C(t)) = \mathbb{E} \left[C(t) \frac{d\mu_C}{d\mu}(t) e^{-\int_0^t \tilde{r}(s) ds} \right]$ we can find

the discount rate functional implicit in the financial engineering model

$$dR[C](s) = -d \ln \left\{ \frac{d\mu_C}{d\mu}(t) e^{-\int_0^t \tilde{r}(s) ds} \right\}_{t=s}$$

$$= -d \ln \frac{d\mu_C}{d\mu}(s) + \tilde{r}(s)$$

THE ACTUARIAL DISCOMFORT

- In the simplest financial engineering examples,

$$\frac{d\mu_C}{d\mu}(t) = e^{-\int_0^t \Theta(s) dW(s) - \frac{1}{2} \int_0^t \Theta^2(s) ds} \quad \text{where } \Theta(s) \text{ is called "the market price of risk"}$$

- This gives a discount rate functional $dR[C](s) = \Theta(s) dW(s) + \left[\frac{1}{2} \Theta^2(s) + \tilde{r}(s) \right] dt$ where the Brownian component clearly could make the entire discount rate functional negative for some values of s on some paths.

STILL A PROBLEM FOR STERN

- With reference to the Stern Review, it's worth noting that in the financial engineering framework the bounded variation component (Stern's δ) must be increased whenever a Brownian component is present.