$$i^j \widehat{f_{X-a}}^{(j)}(ih) = \lim_{M\to\infty} \sum_{m=0}^{M-j} \frac{i^{j+m}}{m!} \widehat{f_{X-a}}^{(j+m)}(0)\, h^m, \text{ which expands to}$$

$$i^j \widehat{f_{X-a}}^{(j)}(ih) = \lim_{M\to\infty} \sum_{m=0}^{M-j} \frac{1}{m!} \left[\sum_{k=0}^{j+m} \frac{(j+m)!}{k!\,(j+m-k)!} (-1)^{j+m-k} a^{j+m-k} i^k \widehat{f_X}^{(k)}(0) \right] h^m$$

$$\text{or } i^j \widehat{f_{X-a}}^{(j)}(ih) = \sum_{l=0}^{j} \frac{1}{(j-l)!} i^{j-l} \widehat{f_X}^{(j-l)}(0) \lim_{M\to\infty} \sum_{m=0}^{M-j} \frac{(j+m)!}{m!\,(m+l)!} (-1)^{m+l} a^{m+l} h^m$$
$$+ \lim_{M\to\infty} \sum_{l=1}^{M-j} \frac{1}{(j+l)!} i^{j+l} \widehat{f_X}^{(j+l)}(0) \sum_{m=l}^{M-j} \frac{(j+m)!}{m!\,(m-l)!} (-1)^{m-l} a^{m-l} h^m$$

We expect this approach to be more accurate for small values of j than for large values. This needs to be tested.

An alternative expansion takes advantage of the duality between translation and exponential tilting.

$$\begin{aligned}
i^j \widehat{f_{X-a}}^{(j)}(ih) &= i^j \frac{d^j}{dt^j} \widehat{f_{X-a}}(t)\,|_{t=ih} \\
&= i^j \frac{d^j}{dt^j} \left[e^{iat} \widehat{f_X}(t) \right] |_{t=ih} \\
&= i^j \sum_{k=0}^{j} \frac{j!}{k!\,(j-k)!} (ia)^{j-k} e^{ia(ih)} \widehat{f_X}^{(k)}(ih)\text{, and now expand and simplify} \\
&= e^{-ah} \sum_{k=0}^{j} \frac{j!}{k!\,(j-k)!} (-1)^{j-k} a^{j-k} \lim_{M\to\infty} \sum_{m=0}^{M-k} \frac{1}{m!} i^{k+m} \widehat{f_X}^{(k+m)}(0)\, h^m
\end{aligned}$$

$$i^j \widehat{f_{X-a}}^{(j)}(ih) = e^{-ah} \lim_{M\to\infty} \left[\sum_{m=0}^{M-j} \frac{1}{m!} \left(\sum_{k=0}^{j} \frac{j!}{k!\,(j-k)!} (-1)^{j-k} a^{j-k} i^{k+m} \widehat{f_X}^{(k+m)}(0) \right) h^m \right.$$
$$\left. + \sum_{m=M-j+1}^{M} \frac{1}{m!} \left(\sum_{k=0}^{M-m} \frac{j!}{k!\,(j-k)!} (-1)^{j-k} a^{j-k} i^{k+m} \widehat{f_X}^{(k+m)}(0) \right) h^m \right]$$

$$\text{or } i^j \widehat{f_{X-a}}^{(j)}(ih) = e^{-ah} \left[\sum_{l=0}^{j} i^{j-l} \widehat{f_X}^{(j-l)}(0) \sum_{m=0}^{j-l} \frac{j!}{(j-m-l)!\,(m+l)!m!} (-1)^{m+l} a^{m+l} h^m \right.$$
$$\left. + \lim_{M\to\infty} \sum_{l=1}^{M-j} i^{j+l} \widehat{f_X}^{(j+l)}(0) \sum_{m=l}^{j+l} \frac{j!}{(j-m+l)!\,(m-l)!m!} (-1)^{m-l} a^{m-l} h^m \right]$$

We expect this approach to be more accurate for large values of j than for small ones. This needs to be tested.