

**Math 5621**  
**Financial Mathematics II**  
**Final Exam**  
May 1, 2009

This is a take-home exam due back no later than 5 pm on May 6. You can leave it in my faculty mailbox, slip it under my office door if I am not in, or email your solutions to me at [bridgeman@math.uconn.edu](mailto:bridgeman@math.uconn.edu). You can use your notes and/or textbooks as well as any other published source freely during this exam but do not consult with any other person. (The consequence of collaboration will be failure). Please show all your supporting work on attached pages.

1. The risk free rate is 5%. Build a 5-step binomial tree (i.e.  $N=5$ ) using an annualized volatility  $\sigma=.25$ , a terminal time  $T=2$ , risk neutral probabilities  $p(\text{up})=p(\text{down})=1/2$  and a starting value  $S_0=\$20$  for the underlying asset. If  $V$  is a European put option with strike price  $\$21$  expiring at  $T=2$ :
  - a. What is the value of  $V$  at time  $t=0$ ?
  - b. At time  $t=.8$  what is the value of the position in the underlying asset held in the replicating portfolio at the down-up node of the tree?
  - c. If  $W$  is an American put option with strike price  $\$21$  expiring at  $T=2$  on the same underlying asset, what is the value of  $W$  at time  $t=0$ ?
2. Over the past 60 months stock A had an average monthly total return of 1.1%, a standard deviation of monthly total return of 5.8%, and a correlation coefficient 0.5 of its monthly total return with the monthly total return of the market. In the same period, stock B had an average monthly total return of 1.0%, a standard deviation of monthly total return of 5.8%, and a correlation coefficient 0.6 of its monthly total return with the monthly total return of the market. The monthly total return of the market over the same period averaged 0.75 % with a standard deviation of 7.1%. This month the market had a total return (a loss) of (5)%, stock A had a total return of 0% and stock B had a total return (a loss) of (4)%. What was the abnormal return this month for stock A and stock B?:

A\_\_\_\_\_ B\_\_\_\_\_

3. Suppose that the current price in the market for blank silicon wafers used as raw material for chip manufacturing is \$5 per wafer. Your engineering staff tell you that their best and most reliable consultants forecast that the price of blank silicon wafers will rise at an average rate of 12% per year for the next 3 years, 6% per year for the following 5 years, and reach long run equilibrium at 3% annual increase thereafter forever. You think that the forecast makes a lot of sense. You expect to be using 40,000 blank silicon wafers per year in your manufacturing operation for each of the next 20 years. Assume that blank silicon wafers have a

$\beta$  of 0, that the risk-free rate is 4% per year forever, and that any excess stock of silicon wafers from year to year can be stored for a negligible cost. For each of the next 20 years you have purchased a European call option expiring at the end of that year on 40,000 blank silicon wafers with a strike or exercise price of \$7 per wafer. For each of the next 20 years you have taken a short position in a European put option expiring at the end of that year on 40,000 blank silicon wafers with a strike or exercise price of \$7 per wafer. What is the value *today* of your net position in all of these options?

4. The risk-free rate is 2%. Portfolio A has an expected return of 11% and a standard deviation of return 49%. Portfolio B has an expected return of 8% and a standard deviation of return 16%.
- a. From a risk-reward perspective which portfolio do you prefer, and why?

A \_\_\_\_\_ B \_\_\_\_\_

Why?

- b. The two portfolio returns have a correlation of 0.60. What is the optimal allocation ratio to each of A and B in a new portfolio constructed as a combination of A and B?

A \_\_\_\_\_ B \_\_\_\_\_

- c. Which do you prefer, the combined portfolio or your choice in a.? Why?

5. Gimmel Inc. has a beta of 0.5 on its equity, 40% debt in its capital structure, with the debt being valued by the market as essentially risk-free at a 6% pre-tax annual yield. The expected return on the entire market is 18%. Gimmel is considering a project called Gamma to develop a chain of high-end urban retail outlets for its products that it expects will yield 25% annually on an after-tax basis. The main competitor will be Himmel Inc. which ought to have about the same risk characteristics as Gamma. Himmel's equity beta is 2.2 and it has 10% debt in its capital structure. Assume that the marginal tax rate for both companies is 50% and that the Gamma project will be funded with 40% debt, 60% retained earnings. From a purely financial point of view, should Gimmel proceed with the Gamma project? Give specific financial analyses and reasons.

Yes\_\_\_\_\_No\_\_\_\_\_

Why?

6. Hannibal Inc., with a WAAC of 16.65%, is growing both its earnings and its dividends at 5.55% % per year. Assume that it can do that forever. Scipio Inc., with a WACC of 7%, is growing both its earnings and its dividends at 2.22% per year. Assume it can do that forever. The two companies have exactly the same values for assets, earnings and dividends this year. Can you tell whether Hannibal or Scipio has the higher PVGO as % of its total market capitalization? Why or why not? Explain your conclusion with specific formula(s). (There might be more than one correct explanation ... you only need to give one.)

Yes\_\_\_\_Hannibal\_\_\_\_Scipio\_\_\_\_

No\_\_\_\_\_

Explanation:

7. Assume that you believe the basic premises of the Pecking Order Theory for capital structure. Despite that belief, explain why it still might make sense for a company to take on (borrow) new long term debt to finance a project even though it has enough cash and marketable securities easily to finance the project without borrowing. Use at least one formula or diagram to illustrate or support your reasoning.