Sample Exam 2 **Math 2100, Spring 09**

Name:______________________________ Section:__________

1. Let a function in $R^2(x,y)$ be given as follows:

$$f(x,y) = \begin{cases} \dfrac{x^2-y^2}{x^2+y^2}, & \text{if } (x,y) \neq (0,0)\ ; \\ 0, & \text{if } (x,y) = (0,0)\ . \end{cases}$$

Find the largest domain in which the function is continuous. Especially, pay attention to the origin $(0,0)$. **Show all of your work.**

2. Using the Lagrange multiplier method, find the maximum and minimum values of the function $f(x,y) = x^2y$ subject to the constraint $x^2 + 2y^2 = 6$.

3. a) Compute $\int\int_D xy\, dA = \int_0^1 \int_y^{2-y^2} xydxdy$.

b) Sketch the domain D over which the double integral of (a) is computed. Indicate all relevant pieces of information regarding the domain.

c) Reverse the order of the integrals in (a). **Need not to evaluate the integral.**

4. Using polar double integral, find the area of a leaf of the four leaved rose $r = \sin 2\theta$. **Hint: Need to use one of the half angle formulas:** $\cos 2x = 2\cos^2 x - 1 = 1 - 2\sin^2 x$.

5. Given a function $F(x,y,z) = x^2 + 2y^2 + 3z^2$,

a) compute the gradient of F at the general point (x,y,z).

b) compute the directional derivative of F at (1,0,-1) in direction (1,1,1).

c) find the equation of the level surface of the function passing through (4, -1,1).

d) find the equation of the tangent plane to the level surface at (4, -1, 1).

Good Luck!!