
Parametric Curves

Part 1: Parametrizations.

1. In each part of this problem,

- (i) eliminate the parameter to obtain an equation in terms of x and y ,
 - (ii) graph the parametric curve over the indicated range of t -values, including arrows on the curves to indicate the direction of increasing values of t and labeling the points on the curve where t has its first value, its last value, and its middle value. The middle t -value need not correspond to the middle position along the curve.
- (a) $x = \sqrt{t} + 4$, $y = 3\sqrt{t}$ for $0 \leq t \leq 16$.

(b) $x = 3 \cos t$, $y = 3 \sin t$ for $0 \leq t \leq \frac{\pi}{2}$.

2. Write down a parameterization of the following circles and then draw an arrow on the graph indicating how the parametrization traces out the circle. Mark the starting point as $t = 0$.

(a) A circle centered at the origin with radius 16, traversed counterclockwise starting at $(16, 0)$.

(b) A circle centered at $(2, 3)$ with radius 2, traversed clockwise starting at $(2, 5)$.

3. An ellipse centered at the origin has major radius 5 along the x -axis and minor radius 2 along the y -axis.
- (i) Give a parameterization of the ellipse so that $0 \leq t \leq 10$ traces out the ellipse once counterclockwise starting at $(5, 0)$, draw a graph with arrows marking the direction of increasing t on the ellipse, and mark the point where $t = 7$.
 - (ii) Use calculus and the parametrization you found in (i) to compute the slope of the tangent line to the ellipse at the point where $t = 7$, both *exactly* in terms of values of trigonometric functions and as an approximation to two decimal places.

Part 2: Polar Coordinates.

4. In each part below, P is a point in polar coordinates. Do the following:

(i) Convert P to Cartesian coordinates (give exact values, not approximations).

(ii) Plot P in the xy -plane.

(iii) Give two additional representations of P in polar coordinates.

(a) $P = \left(2, \frac{\pi}{4}\right)$

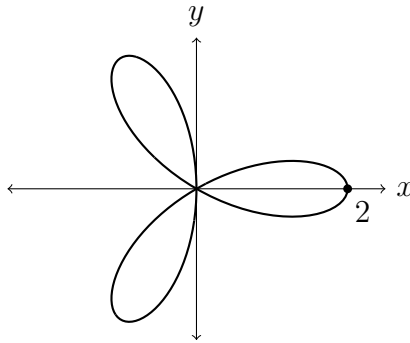
(b) $P = \left(-3, \frac{3\pi}{4}\right)$

(c) $\left(-1, -\frac{\pi}{3}\right)$

5. (a) On separate axes plot the two polar equations $r = 1 + 2 \sin \theta$ and $r = 1 - 2 \sin \theta$, drawing arrows on each curve to indicate the direction traced out as θ increases.

- (b) Use calculus and polar coordinates to find the equation of the tangent line to $r = 1 + 2 \sin \theta$ at the point $(x, y) = (1, 0)$, which you graphed in the previous part. (Hint: On this curve write x and y in terms of θ .)

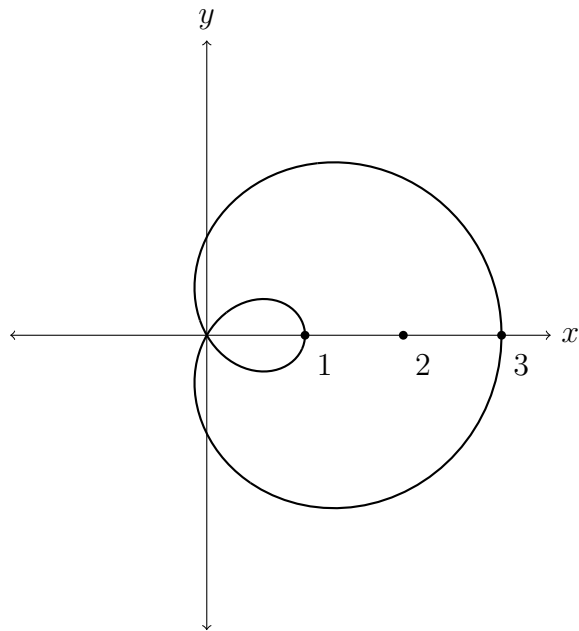
6. Below is the 3-leaf rose $r = 2 \cos(3\theta)$.



- (a) Mark points on the graph where $\theta = 0$ and $\theta = \pi/3$, and draw arrows on the rose to indicate the direction in which it is traced out as θ increases.
- (b) Determine the smallest positive angle θ at which the rose passes through the origin, and use this to help you set up an integral for the area of the right leaf.

- (c) Compute the integral in part b. (Hint: $\cos(2\alpha) = 2 \cos^2 \alpha - 1$, so $\cos^2 \alpha$ can be written in terms of $\cos(2\alpha)$, which is easier to integrate.)

7. Below is the graph of the limaçon $r = 1 + 2 \cos \theta$.



- (a) Mark the points on the curve where θ is $0, \pi/2, \pi, 3\pi/2,$ and 2π and use this to draw arrows on the curve indicating the direction in which it is traced out as θ increases.
- (b) Determine all θ between 0 and 2π where the curve passes through the origin.
- (c) Set up, **but do not evaluate**, an integral for the area of the inner loop of the limaçon entirely in terms of θ . Be sure the bounds of integration are right.