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## Infinite Series II

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**Solutions to these problems should include your work.**

**Part 1: Convergence of Series.**

1. Let  $s = \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{3n+2}$ .

- (a) Write out the first 5 *terms* in this series, as fractions.
- (b) Explain why  $s$  converges using the alternating series test.
- (c) Let  $s_n$  be the  $n$ th partial sum in this series. Use the alternating series estimation theorem to carry out the following two tasks:
  - (i) Give a bound from above on  $|s - s_{100}|$ , giving your answer to four digits after the decimal point.
  - (ii) Find an integer  $n$  such that  $|s - s_n| \leq 1/10000$ .

2. Use any convergence test to determine whether the following series converge or diverge. In each case you should state the test you are using, check the hypotheses for that test are satisfied (e.g., for the integral test with  $a_n = f(n)$ , check  $f(x)$  is decreasing), carry out the test with clear explanations, and then give the conclusion from the test.

(a) 
$$\sum_{n=0}^{\infty} \frac{1}{n^2 - 5n + 7}.$$

(b) 
$$\sum_{n=1}^{\infty} \frac{\cos n}{n^3}.$$

(c) 
$$\sum_{n=2}^{\infty} \frac{\ln n}{n}.$$

(d) 
$$\sum_{n=0}^{\infty} \frac{n^2}{8^n}.$$

**Part 2: Power Series.**

3. Determine the radius of convergence and interval of convergence for the following power series. When determining the interval of convergence, be sure to check *both* endpoints of the interval. Explain what convergence test you use as the first step in applying any test.

(a) 
$$\sum_{n=0}^{\infty} \frac{x^n}{7^{n+1}}$$

(b) 
$$\sum_{n=0}^{\infty} \frac{nx^n}{(2n+1)!}$$

(c) 
$$\sum_{n=1}^{\infty} \frac{x^{2n+1}}{3^n n^2}$$

4. Use the known power series  $\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n$  when  $|x| < 1$  to find the power series at  $x = 0$  for the following related functions *without* using Taylor's formula for coefficients. Find the interval of convergence in all cases.

(a)  $\frac{1}{1-x^3}$

(b)  $\frac{1}{2-5x}$  Hint: Write  $\frac{1}{2-5x} = \frac{1}{2} \cdot \frac{1}{1-?}$ .

(c)  $\frac{1}{(1-x)^3}$  Hint: Differentiate twice.