Math 1132 Worksheet 5 Name ______________ Discussion Section ____

Infinite Series I

Solutions to these problems should include your work.
Part 1: Sequences and Partial Sums.

1. Let $a_n = \dfrac{(-1)^{n-1}}{n}$.

   (a) Plot a_n vs. n for $n = 1, 2, 3, 4, 5, 6, 7, 8$.

   (b) Compute $s_n = \sum_{k=1}^{n} a_n$ for $n = 1, 2, \ldots, 8$ to three digits after the decimal point and then plot s_n vs. n for $n = 1, 2, \ldots, 8$.

n	1	2	3	4	5	6	7	8
s_n								

2. Let $a_n = \dfrac{\sin n}{n}$, where angles are measured in *radians*.

(a) Compute a_n for $n = 1, 2, \ldots, 8$ to three digits after the decimal point and then plot a_n vs. n for $n = 1, 2, \ldots, 8$.

n	1	2	3	4	5	6	7	8
a_n								

(b) Compute $s_n = \sum_{k=1}^{n} a_n$ for $n = 1, 2, \ldots, 8$ to three digits after the decimal point and then plot s_n vs. n for $n = 1, 2, \ldots, 8$.

n	1	2	3	4	5	6	7	8
s_n								

Part 2: Limits of Sequences.

3. For each of the following recurrence relations compute a_n for $n = 0, 1, \ldots, 5$ and then find an explicit formula for a_n in terms of n.

   (a) $a_{n+1} = a_n - 2$ where $a_0 = 3$.

   (b) $a_{n+1} = 2 - a_n$ where $a_0 = 0$.

   (c) $a_{n+1} = 3a_n$ where $a_0 = 2$.

4. Determine the limit of the sequence or state the limit does not exist. If there is a limit, show the calculations that explain how you are finding the limit.

(a) $\left\{\dfrac{n^4}{n^4+1}\right\}$ for $n \geq 1$.

(b) $\left\{\dfrac{\cos(\pi n)}{\sqrt{n}}\right\}$ for $n \geq 1$.

(c) $\left\{\dfrac{\cos n}{\sqrt{n}}\right\}$ for $n \geq 1$.

(d) $\left\{\sqrt{\left(1+\frac{1}{2n}\right)^n}\right\}$ for $n \geq 1$.

5. Is the sequence $\{2^n 3^{-n}\}$ for $n \geq 0$ increasing, decreasing, or neither? Is it bounded above or below? If it converges, give the limit.

Part 3: Infinite Series.

6. Compute the following geometric series *exactly*. Write "divergent" if it diverges.

(a) $\displaystyle\sum_{n=2}^{\infty} \left(\frac{3}{7}\right)^n$

(b) $\displaystyle\sum_{n=0}^{\infty} 5 \cdot \frac{2^{n+1}}{3^{n+2}}$

7. For each of the following infinite series, use the remainder bound based on the integral test to find an n such that the nth partial sum is within a specified distance from the value of the series.

   (a) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^4}$; within .001.

   (b) $\displaystyle\sum_{n=3}^{\infty} \frac{1}{n(\ln n)^2}$; within .1.