

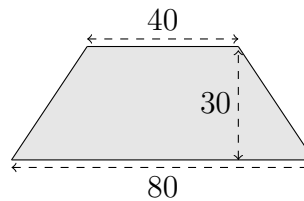
---

# Applications of Integration

---

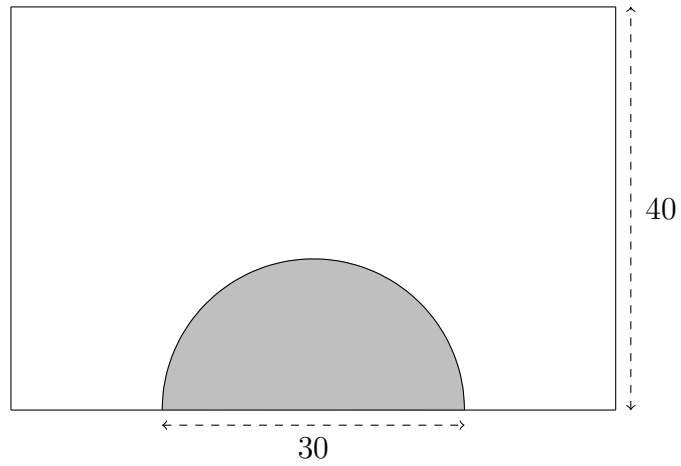
## Part 1: Hydrostatic Force.

1. The wall of a dam is an isosceles trapezoid, measuring 80 meters on the bottom, 40 meters on the top, and being 30 meters tall, as in the diagram below.



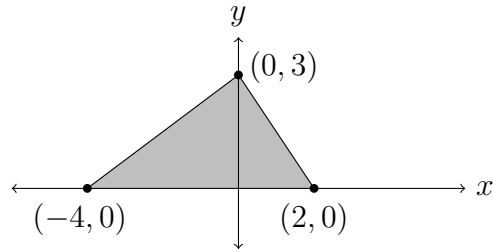
- a) Determine the width  $w(y)$  of the dam wall  $y$  meters from the bottom, for  $0 \leq y \leq 30$  (so  $w(0) = 80$  and  $w(30) = 40$ ).
- b) Set up an integral for the hydrostatic force against the wall, in Newtons, when the water reaches the top of the dam wall. Then use calculus to compute the integral, giving your final answer to the nearest integer. You may use  $g = 9.8 \text{ m/sec}^2$  for the acceleration due to gravity in metric units.
- c) Set up, but **do not evaluate**, a definite integral for the hydrostatic force against the wall when the water level is 20 meters deep. Explain the changes from parts a and b before you write down the integral.

2. An aquarium pool that is 40 feet deep has a semi-circular viewing window at the bottom with a diameter of 30 feet. Express the hydrostatic force of water against this window in pounds as a definite integral, but **do not** evaluate the integral.

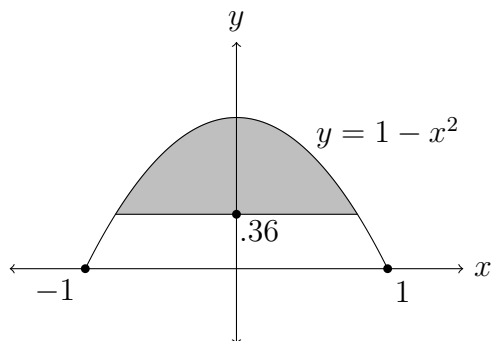


**Part 2: Center of Mass.**

3. Determine the center of mass (centroid) of the shaded triangular region below. Give both coordinates  $(\bar{x}, \bar{y})$  exactly.

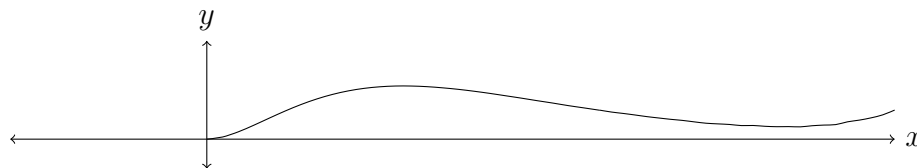


4. The shaded region below is bounded by the upside-down parabola  $y = 1 - x^2$  and a straight line at height .36. Its centroid has  $\bar{x} = 0$  by symmetry. Use integration to determine  $\bar{y}$ , and give your final answer to the nearest thousandth.



**Part 3: Probability.**

5.



a) Find an antiderivative of  $x^2 e^{-x}$  using integration by parts and use that to determine a constant  $k$  that makes

$$f(x) = \begin{cases} 0 & \text{if } x \leq 0, \\ kx^2 e^{-x}, & \text{if } x \geq 0 \end{cases}$$

a probability density function. (The graph of  $y = x^2 e^{-x}$  for  $x \geq 0$  is shown above.)

b) If a continuous random variable  $X$  has probability density function  $f(x)$  above, compute the probability that  $X \geq 3$  both exactly and then to the nearest hundredth.

c) Find the mean of the probability density function in part a.