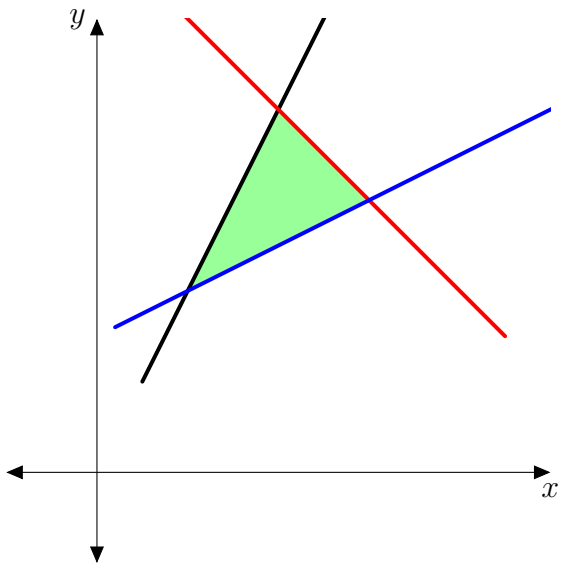
Math 1132 Worksheet 2 Name _____ Discussion Section _____

Area between Curves, Volumes, and Work

Solutions to these problems should show all of your work, not just a single final answer. All areas are nice numbers, not nasty decimals.

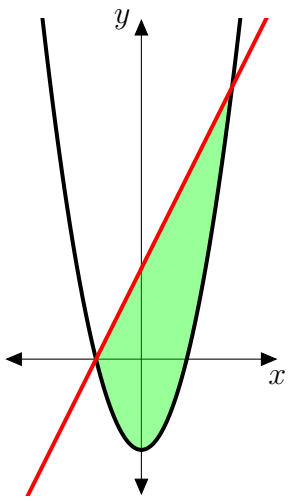
1. We want to find the area of the triangle with vertices $(1, 2)$, $(2, 4)$, and $(3, 3)$, by calculus.



(a) Find equations for the three lines connecting the vertices.

(b) Use the equations in part a to express the area of the triangle in terms of integrals, and then compute the area.

2. We want to find the area of the region bounded by $y = 2x + 4$ and $y = x^2 - 4$.



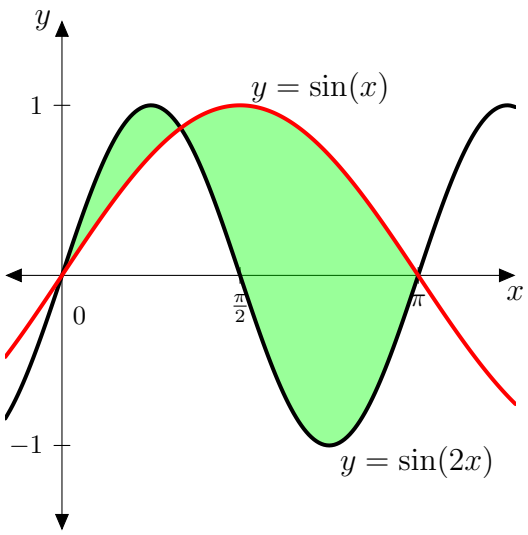
(a) Determine the coordinates of the points where the line and parabola intersect.

(b) Express the area as an integral with respect to x.

(c) Express the area as an integral with respect to y.

(d) Explain which of (a) or (b) is simpler to compute, and use the simpler one to find the area.

3. Find the area of the regions between $y = \sin x$ and $y = \sin(2x)$ for $0 \leq x \leq \pi$. (Hint: To find the exact coordinates of the point where the graphs cross, recall that $\sin(2x) = 2\sin x \cos x$.)



4. A solid region has a circular base of radius 3 whose cross-sections perpendicular to the base and parallel to the y-axis are equilateral triangles.

   (a) Placing the circular base in the plane so it's centered at the origin, determine the side length of the cross-sectional triangle that passes through $(x, 0)$, for $-3 \leq x \leq 3$. (Your final answer will depend on x.) Draw a clear diagram in your solution.

   (b) Set up, but **do not evaluate**, an integral equal to the volume of this solid region. (Hint: the area of an equilateral triangle with side length s is $\frac{s^2}{4}\sqrt{3}$.)

5. A circular swimming pool with diameter 8 m. and height 1.5 m. contains water to a depth of 1 m. Compute the amount of work required to pump all the water out of the pool over the side, giving your final answer in joules to the nearest integer. Consider the density of water to be 1000 kg/m^3.

6. A cable that weighs 4 lb/ft is used to lift 300 lbs of coal up a mineshaft that is 1000 ft deep. Determine the work needed to bring the coal up the mineshaft by means of the cable. (Hint: Compute the work done in lifting the cable and the coal separately.)