Math 1132 Worksheet 1 Name ______________________ Discussion Section ____

Integration by Substitution (Review)

Part 1: Compute the indefinite integral in three steps: (1) specify the substitution $u =?$ and $du =?$, (2) rewrite the integral *completely* in terms of the new variable u, (3) compute the new integral and express the final answer in terms of the original variable.

Example. Evaluate $\int \frac{x}{x^2+1}\,dx$.

Solution.

(1) Set $u = x^2 + 1$, so $du = 2x\,dx$.

(2) Since $x\,dx = \frac{1}{2}du$, we have $\int \frac{x}{x^2+1}dx = \int \frac{(1/2)du}{u} = \int \frac{1}{2} \cdot \frac{1}{u}\,du$.

(3) $\int \frac{1}{2} \cdot \frac{1}{u}\,du = \frac{1}{2}\ln|u| + C = \frac{1}{2}\ln|x^2+1| + C = \frac{1}{2}\ln(x^2+1) + C$.

1. Evaluate $\int x^2 \sin(x^3)\,dx$.

2. Evaluate $\int x\sqrt{4x+1}\,dx$.

3. Evaluate $\int \frac{1+x}{1-x}\,dx$.

4. Evaluate $\int \frac{1}{x \ln x}\,dx$.

5. Evaluate $\int \sin^3 x\,dx$. (Hint: $\sin^2 x = 1 - \cos^2 x$.)

Part 2: Express the definite integral in x as a new definite integral in the new variable u. Don't evaluate the new definite integral.

Example. Rewrite $\int_2^3 xe^{-x^2}\,dx$ in terms of $u = x^2$.

Solution. When $u = x^2$, $du = 2x\,dx$. If $x = 2$ then $u = 4$, and if $x = 3$ then $u = 9$, so $\int_2^3 xe^{-x^2}\,dx = \int_{u=4}^{u=9} e^{-u}\frac{du}{2} = \frac{1}{2}\int_4^9 e^{-u}\,du$.

6. Rewrite $\int_0^1 (3x+1)^2\,dx$ in terms of $u = 3x+1$.

7. Rewrite $\int_0^1 x^2(1+2x^3)^5\,dx$ in terms of $u = 1+2x^3$.

8. Rewrite $\int_0^{\pi/3} \frac{\sin x}{\cos^2 x}\,dx$ in terms of $u = \cos x$.

9. Rewrite $\int_0^{\pi/3} \sin x\cos x\,dx$ in terms of $u = \cos x$.

10. Rewrite $\int_0^{\pi/3} \sin x\cos x\,dx$ in terms of $u = \sin x$.