1. If the statement is always true, circle the printed capital T. If the statement is sometimes false, circle the printed capital F. In each case, write a careful and clear justification or a counterexample.

   (a) If the n^{th} partial sum of a series $\sum_{n=1}^{\infty} a_n$ is $s_n = 1 + \frac{n}{3^n}$ then $a_n = \frac{2-n}{3^n}$ for $n > 1$: (a) T F

   (b) The geometric series $\sum_{n=4}^{\infty} (\frac{1}{3})^n$ converges to $\frac{3}{2}$. (b) T F

   (c) If $\lim_{n\to\infty} a_n = 0$ then the series $\sum_{n=1}^{\infty} a_n$ converges. (c) T F

   (d) The series $\sum_{k=1}^{\infty} \frac{(-1)^k}{k^3}$ converges conditionally. (d) T F

   (e) If $\sum_{n=1}^{\infty} |a_n|$ diverges then $\sum_{n=1}^{\infty} a_n$ diverges. (e) T F

   (f) The sequence $a_n = \frac{\ln(2n)}{\ln(n)}$ converges to 1. (f) T F

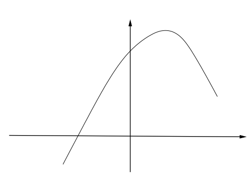
   (g) If the power series $\sum_{k=0}^{\infty} a_k\,(x-4)^k$ has a radius of convergence equal to 2 then $\sum_{k=0}^{\infty} a_k$ diverges. (g) T F

2. For each multiple choice question, circle the correct answer. There is only one correct choice for each answer. No justification is required.

   (a) Which of the following sequences is both bounded and monotonic?

   (a) $\{a_n\} = \{n^2\}_0^{\infty}$
   (b) $\{a_n\} = \{\frac{n}{n+1}\}_0^{\infty}$
   (c) $\{a_n\} = \{\frac{\sin(\pi n)}{n}\}_0^{\infty}$
   (d) $\{a_n\} = \{\frac{n}{\sqrt{n+1}}\}_0^{\infty}$
   (e) None of the above

   (b) The function $f(x)$, whose graph is shown, has the Taylor polynomial of degree 2 centered at $x = 0$ given by $p_2(x) = a + bx + cx^2$. What can you say about a, b, c?

   (i) a is: negative, zero or positive
   (ii) b is: negative, zero or positive
   (iii) c is: negative, zero or positive

   (c) The value of the telescoping series $\sum_{k=1}^{\infty} (\frac{1}{k} - \frac{1}{k+1})$ is

   (a) 0 (b) 1 (c) 2 (d) 1/2 (e) None of the above

3. Consider the following series, all of which converge. For which of these series do you get a conclusive answer when using the **Ratio Test** to check for convergence? Write the letters of all possible answers. If no series satisfies this condition, write"none". You do not need to show your work.

A $\displaystyle\sum_{k=1}^{\infty} \frac{k^3}{2k^5+k^2+1}$ **B** $\displaystyle\sum_{k=1}^{\infty} \frac{k^6}{k!}$ **C** $\displaystyle\sum_{k=1}^{\infty} (3k+4)^{-k}$

D $\displaystyle\sum_{k=1}^{\infty} \frac{\ln k}{k^2}$ **E** $\displaystyle\sum_{k=1}^{\infty} (-1)^k \frac{2}{5^k}$

4. Consider the following sequences and answer the questions that follow by circling all that apply.

(a) $\displaystyle\left\{a_n = \left(\frac{1-2n}{n+1}\right)^2\right\}_{n=1}^{\infty}$

(b) $\left\{b_n = 3^{n+5}2^{-n}\right\}_{n=1}^{\infty}$

(c) $\displaystyle c_n = \left\{\frac{(-5)^{n+1}}{(3)^n}\right\}$

(s) $\displaystyle S_n = \sum_{k=2}^{n} \frac{k}{k^3-2}$

(a) Which of the above sequences are bounded? (a) a_n (b) b_n (c) c_n (s) S_n [3]

(b) Which of the above sequences are increasing? (a) a_n (b) b_n (c) c_n (s) S_n [3]

(c) Which of the above sequences are convergent? (a) a_n (b) b_n (c) c_n (s) S_n [3]

5. For each of the following

(i) Find the Taylor polynomial of order 3 generated by f and centered at a.

(ii) Use the 3rd order Taylor polynomial to approximate the given value of f.

(iii) Use the remainder term to find an upper bound for the absolute error in this approximation.

(a) $f(x) = \sqrt{x}$, $a = 16$. Approximate $f(16.2)$.

(b) $f(x) = \cos x$, $a = \frac{\pi}{4}$. Approximate $f(\frac{3\pi}{16})$.

(c) $f(x) = e^{-2x}$ centered at $a = 0$. Approximate $f(0.1)$

6. (a) Find the Taylor series of $f(x) = e^{-x^2}$ centered at 0.

   (b) Evaluate $\int e^{-x^2}\,dx$ as an infinite series.

   (c) Use part (a) to find the Taylor series of $f(x) = 2xe^{-x^2}$ centered at 0.

7. Determine whether the following series converge conditionally, converge absolutely or diverge. Show your work in applying any tests used.

   (a) $\displaystyle\sum_{k=1}^{\infty} \frac{\sqrt{k^2+1}}{k}$ (b) $\displaystyle\sum_{k=2}^{\infty} \frac{1}{k\ln k}$ (c) $\displaystyle\sum_{k=2}^{\infty} ke^{-2k^2}$

   (d) $\displaystyle\sum_{k=0}^{\infty} \frac{4+3^k}{4^k}$ (e) $\displaystyle\sum_{k=2}^{\infty} \frac{1}{k(\ln k)^3}$ (f) $\displaystyle\sum_{k=1}^{\infty} \left(\frac{k+1}{2k}\right)^k$

   (g) $\displaystyle\sum_{k=1}^{\infty} \frac{k^4}{e^{3k}}$ (h) $\displaystyle\sum_{k=2}^{\infty} \frac{(-1)^k}{k^2-1}$ (i) $\displaystyle\sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{\sqrt{k^2+1}}$

8. Find the interval of convergence of the power series $\displaystyle\sum_{n=1}^{\infty} \frac{1}{2^n n}(x-2)^n$

9. Consider the series $\displaystyle\sum_{n=1}^{\infty} \frac{3^n}{n}x^n$

   (a) Find the radius of convergence, R, for this series.
   (b) Find the interval of convergence.
   (c) For which x does this series converge absolutely?

10. The infinite series $\displaystyle\sum_{n=1}^{\infty}(-1)^n\frac{n^2}{2^n}(x-1)^n$ has a radius of convergence R = 2.

    (a) Explain carefully what the "radius of convergence" tells us about the series.
    (b) Find the interval of convergence. Show step-by-step work.

11. How many terms of the series The infinite series $\displaystyle\sum_{n=1}^{\infty}(-1)^{n+1}\frac{1}{7n+5}$ do we need to add in order to find the sum of the series to with an accuracy of 0.00001 (that is, $|\text{error}| < 0.00001$)?

12. Approximate the sum of the series $\displaystyle\sum_{n=1}^{\infty}(-1)^{n+1}\frac{1}{n^2+1}$ correct to three decimal places.