University of Connecticut
Department of Mathematics

MATH 1132 | PRACTICE EXAM 1 | SPRING 2015

NAME: ______________________________

Instructor Name: ______________________________ Section: __________

TA Name: ______________________________ Discussion Section: __________

Read This First!

- Please read each question carefully. Show **ALL** work clearly in the space provided. In order to receive full credit on a problem, solution methods must be complete, logical and understandable.
- Answers must be clearly labeled in the spaces provided after each question. Please cross out or fully erase any work that you do not want graded. The point value of each question is indicated after its statement. No books or other references are permitted.
- Calculators are allowed but you must show all your work to receive credit on the problem.
- Except for problems about approximate integration or probability, give any numerical answers in exact form, **not as approximations**. For example, one-third is $\frac{1}{3}$, not .33 or .33333. And one-half of π is $\frac{1}{2}\pi$, not 1.57 or 1.57079.
- **Note**: This is only a sample exam. It illustrates the kinds of topics that will be on the midterm, but is no substitute for a thorough review (including your homework and worksheet problems). In particular, problems on the actual exam as well as the point distribution among those problems will not be identical to what you see in the sample exam.

1. If the statement is always true, circle the printed capital T. If the statement is sometimes false, circle the printed capital F. In each case, write a clear justification or a counterexample.

   (a) A sphere of radius 3 has volume $\int_{-3}^{3} \pi(9 - x^2)\, dx$. (a) T F

   Justification:

   (b) Under Hooke's law, the work required to stretch a spring 2 inches beyond its natural length is twice that required to stretch it 1 inch beyond its natural length (b) T F

   Justification:

   (c) The trapezoid rule with $n = 10$ for $\int_{0}^{4} x^3\, dx$ will be an underestimate of the integral's value. (c) T F

   Justification:

   (d) $\int_{0}^{\pi/2} \tan x\, dx = \sec^2(\pi/2) - \sec^2 0$ (d) T F

   Justification:

   (e) The length of the curve $x = \frac{2}{3}(y-1)^{3/2}$ over the interval $1 \leq y \leq 4$ by is given the definite integral $L = \int_{1}^{4} \sqrt{y}\, dy$. (f) T F

   Justification:

2. Find the following general antiderivatives. Remember to add $+C$ to your final answer.

   (a) $\int x^n \ln x\, dx$, where $n \neq -1$

   (b) $\int \frac{x}{\sqrt{x^2 - 1}}\, dx$

   (c) $\int e^{2x} \cos(4x)\, dx$

   (d) $\int \frac{x^2 - 2}{x^3 + 4x}\, dx$

3. Determine whether each of the following improper integrals is convergent or divergent. For those that are convergent, give their exact value (not decimal approximations). Those determined to be divergent must have an explanation for their divergence.

   (a) $\int_{1}^{\infty} \frac{dx}{\sqrt{x}}$

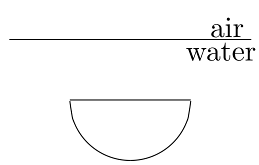
   (b) $\int_{0}^{\infty} \frac{dx}{(2x+1)(x+4)}$

4. Set up, but do **NOT** evaluate, integrals or sums of integrals for the area of the region bounded by the following curves. Do not use absolute value signs in your final answer. On the set of axes in each part draw a picture of the curves and region between them.
   (a) $y = \sin x$ and $y = \cos x$ for $0 \leq x \leq \pi$.
   (b) $y = 2^x$ and $y = 2$ in the first quadrant.

5. Set up, but do **NOT** evaluate, integrals equal to the volumes of the following solid regions. On the set of axes in each part draw a picture of the curves and region between them.
   (a) Revolve the region between $y = x^2$ and $y = x$ in the first quadrant around (a) the y-axis; (b) the line $y = -3$.
   (b) The solid whose base is bounded by $y = x^2 - 1$ and $y = 1 - x^2$ with $-1 \leq x \leq 1$ and whose cross-sections parallel to the y-axis are equilateral triangles.

6. (a) A force of 30 N is required to maintain a spring stretched from its natural length of 12 cm to a length of 15 cm. How much work is done to stretch the spring from 12 cm to 20 cm?
   (b) A cylindrical keg of root beer with height 1 m and diameter $\frac{1}{4}$ m is *half-filled* and is standing upright (on one of its circular bases). The density of the root beer is 2000 kg/m^3. Set up, but do **NOT** evaluate, an integral that is equal to the work required to pump the root beer out of the top of the keg.
   (c) Consider the same half-full cylindrical keg of root beer with height 1 m and diameter $\frac{1}{4}$ m as in part b, but set the keg down on its side. If a hole is made along the (new) top of the rotated keg, set up but do **NOT** evaluate an integral equal to the work required to pump the root beer out. (Hint: Start by drawing a good picture.)

7. A semi-circular plate with *diameter* 1 m is submerged vertically in water with the diameter being the top of the plate, as in the diagram below. Set up but do **NOT** evaluate an integral equal to the hydrostatic force on the plate, in Newtons, if the diameter lies 1 meter below the surface of the water. The density of water is 1000 kg/m^3.



8. For $n > 0$ determine the coordinates of the centroid for the region in the first quadrant bounded by $y = x^n$, the x-axis, and the line $x = 1$. Your answer will depend on n.

9. (a) Use the Trapezoid Rule with $n = 6$ to estimate $\int_0^6 x^2 + 1\,dx$

   (b) Use the error bound formulas on the last page to determine an n such that the trapezoid rule with n subintervals approximates $\int_0^1 \frac{1}{3^x}\,dx$ to within .001.

Trapezoid Rule and Error Bound:

In the formulas below, $a = x_0 < x_1 < x_2 < \cdots < x_{n-1} < x_n = b$ with $x_i - x_{i-1} = \Delta x = \frac{b-a}{n}$ for all i. over $[a, b]$: $|f''(x)| \leq K$ for $a \leq x \leq b$.

$$T_n = (f(x_0) + 2f(x_1) + 2f(x_2) + \cdots + 2f(x_{n-2}) + 2f(x_{n-1}) + f(x_n))\frac{\Delta x}{2}$$

and

$$\left|\int_a^b f(x)\,dx - T_n\right| \leq \frac{K(b-a)}{12}(\Delta x)^2 = \frac{K(b-a)^3}{12n^2},$$

where K is an upper bound on $|f''(x)|$ over $[a, b]$: $|f''(x)| \leq K$ for $a \leq x \leq b$.

10. Find the exact length of the curve.

    (a) $y = 1 + 6x^{3/2}$, $0 \leq x \leq 1$

    (b) $y = \frac{x^3}{3} + \frac{1}{4x}$, $1 \leq x \leq 2$

11. Find the length of the arc of the curve from point P to point Q

    $y = \frac{1}{2}x^2$, $P(-1, \frac{1}{2})$, $Q(1, \frac{1}{2})$