Math 1131 Week 12 Worksheet

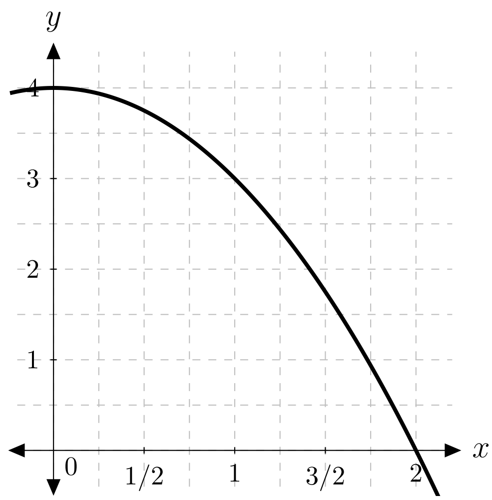
Name: ____________________________

Discussion Section:_______________

Solutions should show all of your work, not just a single final answer.

5.1: Areas and Distances

1. The graph of $y = 4 - x^2$ over the interval $[0, 2]$ is given below.



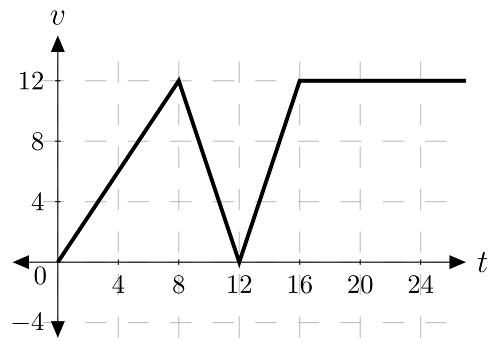
In (a), (b), and (c) below, estimate the area under the graph over $[0, 2]$ using 4 rectangles and the indicated type of endpoints: sketch the rectangles and then compute their areas. Determine from your sketch if the areas of the rectangles provide an underestimate or overestimate of the area under the curve, or if it can't be easily determined.

(a) Right endpoints.

(b) Left endpoints.

(c) Midpoints.

2. Here's a graph of the velocity (in ft/sec) of an object moving along a horizontal line.



(a) Over the interval $0 \leq t \leq 24$, determine the intervals when the object is speeding up and the intervals when the object is slowing down.

(b) From the end of Section 5.1 of the textbook, distance traveled is the area under the velocity vs. time graph. Use this to compute the distance traveled by the object from $t = 0$ to $t = 8$ seconds, in feet.

(c) Compute the distance traveled by the object from $t = 8$ to $t = 20$ seconds, in feet.

5.2: The Definite Integral

3. Evaluate the integral $\int_1^4 (x+1)\,dx$ by drawing a picture and interpreting the integral in terms of areas.

4. Evaluate the integral $\int_{-3}^{3} (1+\sqrt{9-x^2})\,dx$ by drawing a picture and interpreting the integral in terms of areas.

5. T/F (with justification) A definite integral can be negative.

5.3: The Fundamental Theorem of Calculus

6. In worksheet 5.1 you used rectangles to estimate the area under the curve $y = 4 - x^2$ over the interval $[0, 2]$. Use the Fundamental Theorem of Calculus to compute the exact area and then determine if the midpoint approximation from problem 1c in worksheet 5.1 is an underestimate or an overestimate of that area.

7. Use Part 1 of the Fundamental Theorem of Calculus to compute $f'(x)$ in each case below.

   (a) $f(x) = \displaystyle\int_1^x \frac{1}{t^4+1}\, dt$

   (b) $f(x) = \displaystyle\int_x^1 \cos\sqrt{t}\, dt$

   (c) $f(x) = \displaystyle\int_1^{3x} \ln t\, dt$

   (d) $f(x) = \displaystyle\int_0^{x^2} t\sin t\, dt$

8. Evaluate the following definite integrals using the Fundamental Theorem of Calculus.

(a) $\displaystyle\int_2^3 (x^3 + x)\ dx$

(b) $\displaystyle\int_{-1}^1 x^{20}\ dx$

(c) $\displaystyle\int_2^{10} \frac{1}{\sqrt{x}}\ dx$

(d) $\displaystyle\int_0^1 e^{x+1}\ dx$

9. (a) Let $A_0(x) = \int_0^x (1 - t^2)\,dt$, $A_1(x) = \int_1^x (1 - t^2)\,dt$, and $A_2(x) = \int_2^x (1 - t^2)\,dt$. Compute these explicitly in terms of x using Part 2 of the Fundamental Theorem of Calculus.

(b) Over the interval $[0, 2]$, use your answers in part (a) to sketch the graphs of $y = A_0(x)$, $y = A_1(x)$, and $y = A_2(x)$ on the same set of axes.

(c) How are the three graphs in part (a) related to each other? In particular, what does Part 1 of the Fundamental Theorem of Calculus tell you about the graphs in part (a)?

(d) On a graph of $y = 1 - t^2$, for $0 \le t \le 2$, shade the region with signed area $A_0(1.5)$. Indicate with $+$ and $-$ which area counts positively and which negatively.

10. T/F (with justification) The function $F(x) = \int_0^x \cos(t^2)\, dt$ is an antiderivative of $\cos(x^2)$.

11. T/F (with justification) $\int_{-2}^{2} x^{-4}\, dx = \left.\frac{x^{-3}}{-3}\right|_{-2}^{2} = -\frac{1}{12}$.