Section 5.5: Substitution

(1) In this section, we learn the Substitution Rule for integration commonly referred to as "u-substitution." Give multiple examples of functions you would need to use the chain rule to differentiate as well as their derivatives. Then explain how you could recognize the derivative as a function that would require substitution to find the antiderivative of.

Examples of functions you need the chain rule to differentiate as well as their derivatives are:

1) $f(x) = sin(3x^2 + x)$

$$f'(x) = \underbrace{\cos}_{\text{derivative of outside function}} \underbrace{(3x^2 + x)}_{\text{leave inside function alone}} \underbrace{(6x + 1)}_{\text{times derivative of inside function}}$$

$$f'(x) = (6x + 1)\cos(3x^2 + x)$$

2) $f(x) = e^{x^4 - 3x^2 + 5}$

The outer function is the exponential and the inner function is $x^4 - 3x^2 + 5$. Thus,
$f'(x) = e^{x^4 - 3x^2 + 5}(4x^3 - 6x)$

3) $f(x) = ln(x^{-4} + x^4)$

The outside function is the natural log and the inside function is $x^{-4} + x^4$. Thus,
$f'(x) = \frac{1}{x^{-4} + x^4}(-4x^{-5} + 4x^3)$

You could recognize the derivative as a function that would require substitution to find the antiderivative by recognizing the integrand is the product of the derivative of an outer function and the derivative of the inner function.

(2) When evaluating an integral, how do you know you need substitution how do know what is a good candidate for u?

In general, this method works when we have an integral such that $\int f(g(x))g'(x)dx$. With the substitution rule we are replacing a complicated integral by one that is simpler.

Ideas to think about when choosing a u:

a) a function whose derivative (except for perhaps a constant) also appears in the integrand

b) a function that is the inner function of a composition of functions

c) a function that is raised to the highest power

d) a function that appears in the denominator

Try to choose u to be a function in the integrand whose differential also occurs(excluding constant factors). If this can't be done, try choosing u to be the more complicated part of the integrand (possibly the inner function of a composite function).

(3) What considerations do you need to make when using substitution on a definite integral?

Two methods can be used:
1)Evaluate the indefinite integral and then use the fundamental Theorem.
Example from textbook (pg.416)

$$\int_0^4 \sqrt{2x+1}\, dx = \int \sqrt{2x+1}\, dx\Big]_0^4$$

$$= \tfrac{1}{3}(2x+1)^{3/2}\Big]_0^4 = \tfrac{1}{3}(9)^{3/2} - \tfrac{1}{3}(1)^{3/2}$$

$$= \tfrac{1}{3}(27-1) = \tfrac{26}{3}$$

2)Change the integration, when the variable is changed.

If g' is continuous on $[a, b]$ and f is continuous on the range of $u = g(x)$, then

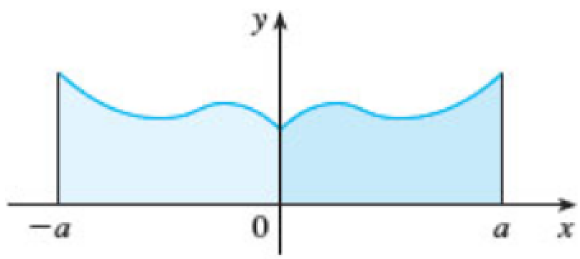
$$\int_a^b f(g(x))\, g'(x)\, dx = \int_{g(a)}^{g(b)} f(u)\, du$$

(4) Give the definitions of even and odd functions and the corresponding rules for integration. Explain with a sketch.

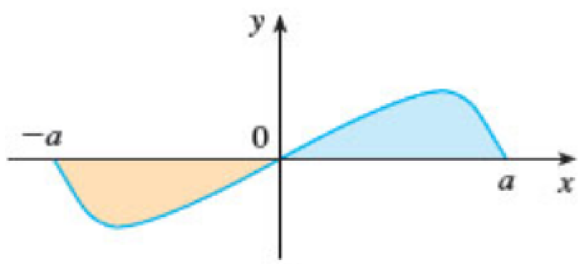
Suppose f is continuous on $[-a, a]$.

(a) If f is even $[f(-x) = f(x)]$ then $\int_{-a}^{a} f(x)\,dx = 2\int_{0}^{a} f(x)\,dx$.

(b) If f is odd $[f(-x) = -f(x)]$ then $\int_{-a}^{a} f(x)\,dx = 0$.



(a) f even, $\int_{-a}^{a} f(x)\,dx = 2\int_{0}^{a} f(x)\,dx$



(b) f odd, $\int_{-a}^{a} f(x)\,dx = 0$

For (a), the area y=f(x) from -a to a is twice the area from 0 to a. For (b) the areas cancel out, thus the integral is zero.

Extra Practice in Book: (it is recommended you practice a lot of substitution until you become comfortable with the method) 5.5: basic examples: 1-36, 53-65, more complicated: 38-48, 66-73, 78, 79, 87, 91