Section 4.9: Antiderivatives

(1) In this section, we learn about antiderivatives. What is an antiderivative of a function?

> If it exists, g is an antiderivative of a function f provided $g' = f$.

(2) Does a function have more than one antiderivative? Why? What is the "most general antiderivative?"

> Yes. Indeed, if $g' = f$, then $(g + C)' = g' + C' = f + 0 = f$ for any constant C.

(3) For each function below, write its antiderivative. For the functions f and g, use F and G, respectively, to represent their antiderivatives.

Function	Most General Antiderivative	Explain in words
$cf(x)$	$cF + C$	c times the antiderivative F, + C.
$f(x) + g(x)$	$F + G + C$	antiderivative of sum is sum of antiderivatives
$x^n (n \neq -1)$	$\frac{1}{n+1}x^{n+1} + C$	antiderivative of x^n is x^{n+1} over $(n + 1)$ + C
$\frac{1}{x}$	$\ln x + C$	antiderivative of $\frac{1}{x}$ is $\ln(x)$ plus C
e^x	$e^x + C$	antiderivative of e^x is e^x plus C
b^x	$\frac{b^x}{\ln b} + C$	antiderivative of b^x is b^x divided by $\ln(b)$ plus C
$\cos(x)$	$\sin(x) + C$	antiderivative of $\cos(x)$ is $\sin(x)$ plus C
$\sin(x)$	$-\cos(x) + C$	antiderivative of $\sin(x)$ is $-\cos(x)$ plus C
$\sec^2(x)$	$\tan(x) + C$	antiderivative of $\sec^2(x)$ is $\tan(x)$ plus C
$\sec x \tan x$	$\sec(x) + C$	antiderivative of $\sec(x)\tan(x)$ is $\sec(x)$ plus C
$\frac{1}{\sqrt{1-x^2}}$	$\sin^{-1}(x) + C$	antiderivative of $\frac{1}{\sqrt{1-x^2}}$ is $\sin^{-1}(x)$ plus C.
$\frac{1}{1+x^2}$	$\tan^{-1}(x) + C$	antiderivative of $\frac{1}{1+x^2}$ is $\tan^{-1}(x)$ plus C

(4) If we are given $f'(x)$, how can we find f? Do we need any additional information to find f exactly? Why?

> Since $\frac{d}{dx}f(x) = f'(x)$, $f(x)$ is an antiderivative of $f'(x)$. Thus, if we know that $F(x) + C$ is the most general antiderivative of $f'(x)$, then $f(x) = F(x) + C$ for some constant C. To find C, one just needs to know $f(x_0)$ for some x_0. Indeed, if $f(x_0)$ is known, then $f(x_0) = F(x_0) + C$, and so $C = F(x_0) - f(x_0)$.

(5) What is the (most general) antiderivative of velocity? Of acceleration? Why?

> Since velocity is the rate of change of position, then position + C, for any constant C, is the (family of) antiderivatives of velocity.
> Since acceleration is the rate of change of velocity, then velocity + C, for any constant C, is the (family of) antiderivatives of acceleration.

Extra Practice in Book: 4.9: 5, 9, 15, 17, 25, 33, 41, 51, 55, 68,