

**Section 2.4: The Precise Definition of a Limit**

- (1) Explain the precise definition of a limit in your own words. What is the role of  $\delta$  and of  $\varepsilon$ ?

- (2) You will be asked to find a  $\delta$  given a specific  $\varepsilon$  in both graph questions and given functions (usually linear). If you are given a specific value for  $\varepsilon$  you should get a specific value for  $\delta$ . If you are asked to do it for a general  $\varepsilon$ , then your answer for  $\delta$  will be in terms of  $\varepsilon$ . Let's practice that in an example. Let  $f(x) = 2x + 1$ . We will show that  $\lim_{x \rightarrow 3} f(x) = 7$

- (a) Fill in the blanks:

For every \_\_\_\_\_ there exists \_\_\_\_\_, such that \_\_\_\_\_  $< \varepsilon$  whenever \_\_\_\_\_  $< \delta$ .

- (b) Let  $\varepsilon = .5$ . Find  $\delta$  so that  $|f(x) - 7| < \varepsilon$  whenever  $|x - 3| < \delta$ . Illustrate with a graph.

- (c) Let  $\varepsilon = .01$ . Find  $\delta$  so that  $|f(x) - 7| < \varepsilon$  whenever  $|x - 3| < \delta$ . Illustrate with a graph.

- (d) Find  $\delta$ (in terms of  $\varepsilon$ ) so that  $|f(x) - 7| < \varepsilon$  whenever  $|x - 3| < \delta$ . Illustrate with a graph.