

Section 6.4: The Normal Distribution

Continuous Probability Distributions: When a random variable is continuous, a probability table or a histogram doesn't make sense since you can't put an entry for every real number. In fact $P(X = x) = 0$ for any one value x of the random variable X !

Example: Let X be the random variable given by the time it takes students to finish an hour exam.

To represent a continuous probability distribution, we use a **probability density function**, or PDF, which is a function whose graph is like a histogram in the discrete case. So areas under the graph correspond to probabilities of the random variable falling in that region.

The Standard Normal Distribution: The random variable Z has a **standard normal distribution** on the interval $(-\infty, \infty)$ if its PDF is given by

$$y = \frac{1}{\sqrt{2\pi}} e^{-0.5x^2},$$

where $\pi \approx 3.14159$ and $e \approx 2.71828$.

Characteristics of the Standard Normal Distribution:

1. The curve is bell shaped.
2. The curve is symmetric about $x = 0$.
3. The curve lies above the x -axis.
4. It approaches, but is never equal to, 0 along both the positive and negative x -axis.
5. The curve is concave down on the interval $(-1, 1)$ and concave up outside this interval.
6. The area under the entire curve is exactly 1.
7. The mean is $\mu = 1$ and the standard deviation is $\sigma = 1$.

Definition: For the standard normal distribution, the area under PDF to left of the number b is denoted $\mathcal{A}(b)$.

Finding Probabilities for Standard Normal Distribution:

Use the table on pages 390-391 in the textbook to find the areas given below.

$$P(Z \leq b) = \mathcal{A}(b)$$

$$P(a \leq Z) = 1 - \mathcal{A}(a)$$

$$P(a \leq Z \leq b) = \mathcal{A}(b) - \mathcal{A}(a)$$

Example 1: Let Z be a random variable with standard normal distribution. Find the following.

(a) $P(Z \leq 0.88)$

(b) $P(-1.27 \leq Z)$

(c) $P(-1.27 \leq Z \leq 0.88)$

The Normal Distribution: The random variable X has a **normal distribution with mean μ and standard deviation σ** on the interval $(-\infty, \infty)$ if its PDF is given by

$$y = \frac{1}{\sigma\sqrt{2\pi}}e^{-0.5\left(\frac{x-\mu}{\sigma}\right)^2}.$$

Characteristics of the Normal Distribution:

1. The curve is bell shaped.
2. The curve is symmetric about $x = \mu$.
3. The curve lies above the x -axis.
4. It approaches, but is never equal to, 0 along both the positive and negative x -axis.
5. The curve is concave down on the interval $(\mu - \sigma, \mu + \sigma)$ and concave up outside this interval.
6. The area under the entire curve is exactly 1.

Finding Probabilities for Normal Distribution: X is random variable with mean μ and standard deviation σ . Then

$$\begin{aligned} P(a \leq X \leq b) &= P\left(\frac{a - \mu}{\sigma} \leq Z \leq \frac{b - \mu}{\sigma}\right) \\ &= \mathcal{A}\left(\frac{b - \mu}{\sigma}\right) - \mathcal{A}\left(\frac{a - \mu}{\sigma}\right). \end{aligned}$$

Example 2: The amount of soda in a 16-ounce can is normally distributed with a mean of 16 ounces and a standard deviation of 0.5 ounces. What percentage of cans will have

- (a) less than 15 ounces?
- (b) more than 17.5 ounces?
- (c) between 15 and 17.5 ounces?

Example 3: Suppose the daily sales at a store is normally distributed with a mean of 500 and a standard deviation of 120. The store wants to give bonuses to the employees on days when the sales are in the top 10% of the daily sales distribution. What is the minimum amount of sales needed to obtain the bonus?