

## Section 6.3: Measures of Spread

**Definitions:** Let  $X$  denote the random variable that takes on the values  $x_1, x_2, \dots, x_n$  with associated probabilities  $p_1, p_2, \dots, p_n$ . If  $\mu = E(X)$ , then the **variance** of the random variable  $X$  is

$$\text{Var}(X) = (x_1 - \mu)^2 p_1 + (x_2 - \mu)^2 p_2 + \cdots + (x_n - \mu)^2 p_n$$

The **standard deviation** of the random variable  $X$  is

$$\sigma(X) = \sqrt{\text{Var}(X)}$$

**Example 1:** Compare the variance and standard deviation for the random variables in the histograms below.

### Another Formula for Variance:

$$\text{Var}(X) = (x_1^2 p_1 + x_2^2 p_2 + \cdots + x_n^2 p_n) - \mu^2$$

**Example 2:** During a five week period in 2007, the stock of a utility company and the stock of a small technology company showed the following weekly percentage changes. Find the variance and standard deviation of the following weekly percentage changes. Which is the riskier stock?

| Company       | Weekly Price Change (%) |      |     |      |      |
|---------------|-------------------------|------|-----|------|------|
| Utility Stock | −1.1                    | 0.4  | 1.5 | −2.5 | −2.2 |
| Tech Stock    | 5.4                     | −1.2 | 5.9 | 1.0  | −3.2 |

**Chebyshev's Inequality:** Let  $X$  be a random variable with expected value  $\mu$  and standard deviation  $\sigma$ . Then the probability that the random variable associated with a random outcome lies between  $\mu - h\sigma$  and  $\mu + h\sigma$  is

$$P(\mu - h\sigma \leq X \leq \mu + h\sigma) \geq 1 - \frac{1}{h^2}, \quad \text{where } h \geq 1.$$

**Example 3:** The expected number of defective parts produced on an assembly line per shift is 40 with a standard deviation of 10. Find the minimum probability using Chebyshev's inequality that the number of defective parts on a particular shift will be between 20 and 60.

**The Variance of the Binomial Distribution:** The variance of the binomial distribution with  $n$  Bernoulli trials with the probability of success  $p$  and the probability of failure  $q$  is given by

$$\text{Var}(X) = npq$$

**Example 4:** On a true-false test with five questions, let  $X$  denote the random variable given by the total number of questions correctly answered by guessing. What is the standard deviation?