

Section 6.2: Measures of Central Tendency

Definition: The **average**, or **mean**, of the n numbers

$x_1, x_2, \dots, x_n$ is

$$\mu = \frac{x_1 + x_2 + \dots + x_n}{n}$$

Note: In statistics, we use μ for the mean when we have all the data in a set. We use $\bar{x}$ to represent the mean when we only have a sample of the data, which is commonly the case in actual studies since you can't get data for the entire population, but rather just a sample of the population.

Example 1: The student ratings of a particular mathematics professor are given in the following table. Find the mean of the student ratings.

[illegible]

Expected Value: Let X denote the random variable that has values $x_1, x_2, \dots, x_n$ with associated probabilities $p_1, p_2, \dots, p_n$. The **expected value**, or **mean**, of the random variable X is

$$E(X) = x_1p_1 + x_2p_2 + \cdots + x_np_n$$

Example 2: In roulette, you can bet on the ball to land on a single number between 1 and 36. There are 38 slots numbered 0, 00, 1, 2, $\dots$, 36. You pay \$1 to place the bet. If you win, you get \$35 and your \$1 bet is returned. If you lose, you lose the \$1 bet. What is your expected return?

Example 3: The SAT test is multiple choice. Each question has 5 possible answers and 1 point is rewarded for each correct answer and $1/4$ point is subtracted for each wrong answer.

- (a) What is the expected value of a random guess?
- (b) What is the expected value of a guess if the test taker knows with certainty that one of the answers is wrong, but has no preference among other 4?

The Expected Value of the Binomial Distribution: The expected value of the binomial distribution with n Bernoulli trials with the probability of success p and the probability of failure q is given by

$$E(X) = np$$

Example 4: On a true-false test with five questions, let X denote the random variable given by the total number of questions correctly answered by guessing. What is the expected value of randomly guessing?

Example 5: Two car dealerships are up for sale. The following two tables give the number of cars sold per day together with the associated probabilities.

	First Dealership			
Number	0	1	2	3
Probability	0.50	0.30	0.15	0.05

	Second Dealership				
Number	0	1	2	3	4
Probability	0.60	0.20	0.05	0.05	0.10

The average profit per car at the first dealership is \$400 and at the second dealership it is \$300. Which dealership will yield the higher daily profit?

Definition: The **median** of an ordered set of numerical data is the middle number if there is an odd number of data points or the average of the middle two numbers if there is an even number of data points.

Definition: The **mode** is the data point that occurs with the most frequency. If every point occurs with equal frequency, we say the set has **no mode**. If there are two data points that occurs with the most frequency, we say the set is **bimodal**.

Example 6: Find the mean, median, and mode of the following set of data.
15, 18, 23, 41, 45, 45, 58, 89

Note: Example 5 on page 306 in the textbook gives an insurance example of expected value that you might find useful for your homework.