

Section 6.1: Random Variables and Histograms

Definition: A **random variable** is a rule that assigns precisely one real number to each outcome of an experiment. When the outcomes are numbers themselves, the random variable is the rule that assigns each number to itself.

Definition: A **probability distribution** is usually written in table form with several rows and columns which lists mutually exclusive events in a sample space and/or the random variable for each event along with the probability associated to each event/random variable. Usually the union of all the events equals the entire sample space so that the sum of the probabilities is 1.

Example 1: A pair of 4-sided dice is tossed. Let X denote the random variable given by the absolute value of the difference of the numbers on the bottom face. Make a probability distribution table for this random variable.

Types of Random Variables:

1. **finite discrete:** assumes only a finite number of values.

Example:

2. **infinite discrete:** assumes an infinite number of values which can be listed by first one, second one, etc.

Example:

3. **continuous:** assumes any of the infinite number of values in an interval of real numbers.

Example:

Histogram: A graphical representation of the data with random variables listed across the bottom and above each random variable is a rectangle with base width 1 and whose height is the frequency or probability for that value of the random variable.

Make a histogram for the probability distribution in Example 1:

Finding probabilities using a histogram: The area of the rectangle in a histogram associated with the random variable X is equal to $P(X)$. Thus the probability that X takes on values in the range $X_i \leq X \leq X_j$ is the sum of the areas of the histogram from X_i to X_j .

Example 2: The student ratings of a particular mathematics professor are given in the following table. Label the random variable, fill in the last line of the table, and draw a histogram.

Event	1	2	3	4	5	6	7	8	9	10
Frequency	5	2	3	0	4	10	10	11	3	2
$P(X = x)$										

What is the probability that the mathematics professor's rating is between 5 and 8 inclusive?

The Binomial Distribution: For a sequence of n Bernoulli trials with the probability of success p and the probability of failure q , the **binomial distribution** is given by

$$P(X = x) = C(n, x)p^x q^{n-x}, \quad \text{for } x = 0, 1, 2, \dots, n.$$

Example 3: On a true-false test with five questions, let X denote the random variable given by the total number of questions correctly answered by guessing. Find the probability distribution and draw a histogram.