

Section 2.3: Inverse of a Square Matrix

Definition: The **inverse** of an $n \times n$ matrix A is a matrix B such that

$$AB = BA = I_n.$$

If B exists, we write $B = A^{-1}$. If a matrix A does not have an inverse, we call A **singular**. If the inverse of A does exist, we call A **nonsingular**.

Example 1: Determine if the following matrices are inverses of each other.

$$A = \begin{bmatrix} -2 & 4 \\ -3 & 7 \end{bmatrix} \text{ and } B = \begin{bmatrix} -7/2 & 2 \\ -3/2 & 1 \end{bmatrix}$$

Finding Inverses

$$\text{Let } A = \begin{bmatrix} 3 & 1 \\ 5 & 2 \end{bmatrix}$$

Guass-Jordan Elimination: Use Guassian elimination to get 1's on the diagonal 0's below and above the 1's.

To find the inverse A^{-1} of a matrix A :

1. Form the augmented matrix $[A|I]$.
2. Use elementary row operations (Guass-Jordan method) to reduce $[A|I]$ to $[I|B]$, if possible.
3. If step 2 is possible, then $B = A^{-1}$. If not, then A is singular.

Elementary Row Operations:

1. **Interchange** the i th row and the j th row.

$$R_i \leftrightarrow R_j$$

2. **Multiply** each member of the i th row by a non-zero constant k .

$$kR_i \rightarrow R_i$$

3. **Replace** each element in the i th row with the corresponding element of the i th row plus k times the j th row.

$$R_i + kR_j \rightarrow R_i$$

Example 2: Find the inverse, if it exists, of $A = \begin{bmatrix} 3 & 1 \\ 5 & 2 \end{bmatrix}$.

Example 3: Find the inverse, if it exists, of $B = \begin{bmatrix} 4 & -2 & 3 \\ 8 & -3 & 5 \\ 7 & -2 & 4 \end{bmatrix}$.

Example 4: Find the inverse, if it exists, of $A = \begin{bmatrix} 1 & 3 & 2 \\ 0 & 1 & 4 \\ 1 & 5 & 10 \end{bmatrix}$.

Example 5: Application to systems of linear equations. Use an inverse of a matrix to solve the following system of equations.

$$4x - 2y + 3z = 3$$

$$8x - 3y + 5z = 2 .$$

$$7x - 2y + 4z = 0$$