

Section 2.2: Matrix Multiplication

Definition 1: A row matrix times a column matrix.

If R is $1 \times n$ and C is $n \times 1$, then

$$RC = \begin{bmatrix} r_1 & r_2 & \cdots & r_n \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ \vdots \\ c_n \end{bmatrix} =$$

Example 1: Compute the following if possible.

$$(a) \begin{bmatrix} 2 & 1 & 0 \end{bmatrix} \begin{bmatrix} -3 \\ 3 \\ 4 \end{bmatrix}$$

$$(b) \begin{bmatrix} 6 & 5 & -1 \end{bmatrix} \begin{bmatrix} 2 \\ 6 \end{bmatrix}$$

Definition 2: A matrix times a matrix.

If A is $m \times p$ and B is $p \times n$, then $AB = D$ where D is the $m \times n$ matrix whose entries d_{ij} are given by the i -th row of A times the j -th column of B .

Example 2: Compute the following if possible.

$$(a) \begin{bmatrix} 2 & 1 & 0 \\ 6 & -4 & 1 \end{bmatrix} \begin{bmatrix} -3 \\ 3 \\ 4 \end{bmatrix}$$

$$(b) \begin{bmatrix} 2 & 0 & 1 & -2 \\ 1 & 3 & -1 & 4 \end{bmatrix} \begin{bmatrix} 2 & 6 & 0 \\ 4 & 1 & 5 \\ -3 & 0 & 4 \\ 1 & 2 & 3 \end{bmatrix}$$

$$(c) \begin{bmatrix} 2 & 0 & 1 & -2 \\ 1 & 3 & -1 & 4 \\ 6 & -8 & 2 & 0 \end{bmatrix} \begin{bmatrix} 2 & 6 & 0 \\ 4 & 1 & 5 \\ -3 & 0 & 4 \end{bmatrix}$$

Example 3: If A has order 5×6 , B has order 6×4 , and C has order 1×6 , are the following matrix products possible? If so, give the order of the product matrix.

(a) AB

(b) BA

(c) AC

(d) CB

More Properties of Matrices

1. $A(BC) = (AB)C$
2. $A(B + C) = AB + AC$
3. $(B + C)A = BA + CA$
4. $AI_n = I_m A = A$, where A is $m \times n$.
5. **Warning!** In general $AB \neq BA$, even if the products are defined.

Example 4: Restaurants. Matrix G shows the number of three types of burritos sold at three local restaurants at lunch. Matrix H shows the price for each type of burrito.

$$G = \begin{array}{cc} & \begin{array}{ccc} \text{veggie} & \text{beef} & \text{chicken} \end{array} \\ \begin{array}{c} \text{store 1} \\ \text{store 2} \\ \text{store 3} \end{array} & \left[\begin{array}{ccc} 15 & 12 & 22 \\ 21 & 14 & 19 \\ 23 & 25 & 17 \end{array} \right] \end{array}$$

$$H = \begin{array}{cc} & \text{price} \\ \begin{array}{c} \text{veggie} \\ \text{beef} \\ \text{chicken} \end{array} & \left[\begin{array}{c} \$3.00 \\ \$4.50 \\ \$4.00 \end{array} \right] \end{array}$$

- (a) Find the product matrix GH .
- (b) What does the matrix GH represent?

Example 5: Application to linear systems. Find matrices A , X , and B ,
so that the following system can be written as $AX = B$.
$$\begin{array}{rrcr} & x & + & z = 2 \\ 2x & + & y & + 4z = 0 \\ & - & y & - z = 1 \end{array}$$