

Section 2.1: Introduction to Matrices

Definitions:

A **matrix** is a rectangular array of numbers.

If A is a matrix with m rows and n columns, then the **order of** A is $m \times n$.

We sometimes write $A = (a_{ij})_{m \times n}$ where a_{ij} denotes the entry in the i -th row and j -th column.

Example 1: A firm has a plant in NYC that has produced the following numbers of items of three products during the four quarters of the last year.

	First Quar- ter	Second Quarter	Third Quar- ter	Fourth Quarter
Product A	250	300	350	300
Product B	300	200	250	150
Product C	100	150	75	100

- Write the production matrix P .
- What is the order of P ?
- What is the p_{23} entry? What does it represent?

Special Types of Matrices

A **row matrix** of dimension n is an $1 \times n$ matrix.

A **column matrix** of dimension m is an $m \times 1$ matrix.

A **square matrix** has order $n \times n$.

The **zero matrix** O is the matrix with every entry equal to 0.

The $n \times n$ **identity matrix** I_n is the matrix with diagonal entries equal to 1 and all other entries equal to 0.

Arithmetic of Matrices

1. **Scalar multiplication:** For a real number (scalar) c , cA is the matrix with every entry of A multiplied by c .
2. **Matrix Addition/Subtraction:** If two matrices A and B have the same order, then $A \pm B = (a_{ij} \pm b_{ij})_{m \times n}$ is the matrix obtained by adding corresponding entries of A and B .
3. **Transpose:** For an $m \times n$ matrix A , the transpose, denoted A^T , is the $n \times m$ matrix whose columns are the rows of A and whose rows are the columns of A .

Example 2: Given $A = \begin{bmatrix} -1 & 3 & 2 \\ 0 & -2 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 3 & 5 \\ 1 & 0 \\ -2 & 6 \end{bmatrix}$, and $C = \begin{bmatrix} 6 & 2 \end{bmatrix}$,

find the following, if possible.

(a) $B - C$

(b) $2A + B^T$

Properties of Matrices

1. $A + O = O + A = A$
2. $A - A = O$
3. $A + B = B + A$
4. $A + (B + C) = (A + B) + C$
5. Two matrices $A = (a_{ij})_{m \times n}$ and $B = (b_{ij})_{m \times n}$ are equal if $a_{ij} = b_{ij}$ for all i and j .

Example 3: Find the values of a, b, c , and d if

$$3 \begin{bmatrix} a & -2 \\ 4 & 1 \end{bmatrix}^T + \begin{bmatrix} -1 & b \\ c & 0 \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ 5 & d \end{bmatrix}.$$