

Section 1.4: Systems of Equations with Non-Unique Solutions

Geometrical Examples for Two Unknowns:

Geometrical Examples for Three Unknowns:

Example 1: Solve the system

$$-2x + y = 3$$

$$-4x + 2y = 10$$

Example 2: Solve the system

$$\begin{aligned}4x - 8y &= 2 \\ -2x + 4y &= -1\end{aligned}$$

Definition: A matrix is in echelon form if:

1. The first non-zero element in any row is 1, called the leading one.
2. The column containing the leading one has all elements below the leading one equal to 0.
3. The leading one in any row is to the left of the leading one in any lower row.
4. Any row consisting of all zeros must be below any row with at least one non-zero element.

Guass Elimination is the process of putting a matrix in echelon form.

Examples of Matrices in Echelon form:

Example 3: Solve the system

$$\begin{array}{rclcl} x & + & y & + & z & = & 1 \\ x & - & 3y & - & z & = & 9 \\ 3x & + & y & + & 2z & = & 7 \end{array}$$

Example 4: Solve the system

$$\begin{array}{rclcrcl} x & + & y & - & z & + & 2u & = & 3 \\ 2x & + & 2y & - & z & + & 3u & = & 8 \\ -4x & - & 4y & + & 8z & - & 12u & = & -4 \end{array}$$

Example 5: Solve the system

$$x + y = 2$$

$$3x - y = 3$$

$$6x + 4y = 1$$