

Section 1.3 Lecture Notes

Creating Systems of Equations:

Step 1: Assign variables to answer the “How many/much?”.

Step 2: Use the information given to determine equations that relate the unknown variables. Often times making a table of the information given will help with this step. In this section, the number of unknowns equals the number of equations, but in general and in Section 1.4 this need not be the case.

Step 3: Solve the system using Gauss Elimination. (More to come on what this is!)

Example 1: Diet planning A dietitian must plan a meal for a patient using two fruits, oranges and strawberries. Each orange contains 1 gram of fiber and 75 mg of vitamin C, while each cup of strawberries contains 2 grams of fiber and 60 mg of vitamin C. How much of each of these fruits needs to be eaten so that a total of 8 grams of fiber and 420 mg of vitamin C will be obtained?

Guass Elimination with two variables on Example 1:

Guass Elimination and Augmented Matrices: For a system of equations, we can write an **augmented matrix** which is a rectangular array of the coefficients of the variables augmented with the constant terms.

For example:

$$\begin{array}{rrcr} x & -3y & +4z & = -4 \\ 3x & -7y & +8z & = -8 \\ -4x & +6y & -z & = 7 \end{array}$$

Goal: Use the elementary row operations to rewrite the augmented matrix so that it has 1's on the main diagonal and 0's below each of these 1's.

Elementary Row Operations:

1. **Interchange** the i th row and the j th row.

$$R_i \leftrightarrow R_j$$

2. **Multiply** each member of the i th row by a non-zero constant k .

$$kR_i \rightarrow R_i$$

3. **Replace** each element in the i th row with the corresponding element of the i th row plus k times the j th row.

$$R_i + kR_j \rightarrow R_i$$

Example 2: Solve the following system of equations using the augmented matrix method and Gauss Elimination.

$$\begin{array}{rrcr} x & -3y & +4z & = & -4 \\ 3x & -7y & +8z & = & -8 \\ -4x & +6y & -z & = & 7 \end{array}$$

Example 2 continued

Example 3: Solve the following system of equations using the augmented matrix method and Gauss Elimination.

$$x + 2y + z - u = -2$$

$$x + 2y + 2z + 2u = 9$$

$$y + z - u = -2$$

$$y - 2z + 3u = 4$$

Example 3 continued

Example 4: Investments Jennifer has \$4200 to invest. She decides to invest in three different companies. The MathOne company costs \$20 per share and pays dividends of \$1.5 per share each year. The NewModule company costs \$60 per share and pays dividends of \$2 per share each year. The JavaTime company costs \$20 per share and pays dividends of \$3 per share each year. Jennifer wants to have three times as many shares of NewModule and JavaTime as of MathOne. Jennifer also wants to earn \$290 in dividends each year. How much should she invest in each company?

Example 4 continued