

Section F.3: Annuities and Sinking Funds

Definitions: An **annuity** is a sequence of equal payments made at regular intervals of time. An **ordinary annuity** is one in which the payments are made at the end of the compounding period. The **term** of an annuity is the time from the beginning of the first period to the end of the last period.

Examples: regular deposits in savings or retirement accounts, monthly mortgage payments, **sinking funds**, which is just an account, often used by corporations, that is established to accumulate funds for some future need.

F = the future value of an ordinary annuity.

R = payment made at the end of each compounding period.

$i = r/m$ is the interest rate per period.

$n = mt$ is the number of payments.

Future Value of an Ordinary Annuity

$$F = R \times \frac{[(1 + i)^n - 1]}{i}$$

Example 1: Finding Future Value A woman pays \$130 at each quarter's end into an account earning 5.4% annual interest compounded quarterly.

(a) How much money will be in the account after 10 years?

(b) How much of this is from deposits? from interest?

$$R = 130 \quad \text{quarterly} \quad m = 4 \quad t = 10 \quad r = 0.054$$

$$m = 4 \quad \therefore i = \frac{0.054}{4} \quad n = 4 \times 10 = 40$$

$$F = \frac{130 \left[\left(1 + \frac{0.054}{4} \right)^{40} - 1 \right]}{\left(\frac{0.054}{4} \right)} = 6835.29$$

$$\text{From deposits} = 130 \times 40 = 5200$$

$$\text{From interest} = 6835.29 - 5200 = 1635.29 \$$$

Example 2: Finding Payment Amount You create a sinking fund that you wish to be worth \$20,000 to use as a down payment on a house at the end of 5 years. The account earns 6.4% interest compounded semiannually.

- (a) How much should you deposit at the end of every 6 months in order to have \$20,000 in 5 years?
- (b) How much interest was earned in the second half of the fourth year?

$$r = 0.064$$

$$m = 2 \text{ (semi annually twice a year)}$$

$$t = 5$$

$$n = 5 \times 2 = 10$$

$$i = \frac{0.064}{2}$$

$$F = 20,000$$

$$R = ?$$

$$F = R \left[\frac{(1+i)^n - 1}{i} \right] = 20,000 = \left[\frac{1 + \left(\frac{0.064}{2} \right)^{10} - 1}{\left(\frac{0.064}{2} \right)} \right] \cdot R$$

$$R = 1728.60 \$$$

F @ the end of 3rd.5 year

$$n = 3 \times 2 + 1 = 7$$

$$\therefore F = 1728.60 \left[\frac{\left(1 + \frac{0.064}{2} \right)^7 - 1}{\left(\frac{0.064}{2} \right)} \right]$$

$$= 13325.82$$

F @ the end of 4 year

$$n = 4 \times 2 = 8$$

$$F = 1728.60 \left[\frac{\left(1 + \frac{0.064}{2} \right)^8 - 1}{\left(\frac{0.064}{2} \right)} \right]$$

$$F = 15480.81$$

$$F_4 - F_{3.5} = 2154.99 \$ \text{ (This includes a payment made in 2nd half of the year)}$$

$$\therefore \text{Int earned} = 2154.99 - 1728.60 = 426.39 \$$$

Finance Mathematics

Future Value of Annuity.

$$F = R \frac{[(1+i)^n - 1]}{i}$$

F = Future value

R = payment made at the end of each compounding period 'Just like Rent'

i = r/m interest rate/period

n = $m \cdot t$ is number of payments.

The idea of this annuity is to save for the future

Examples

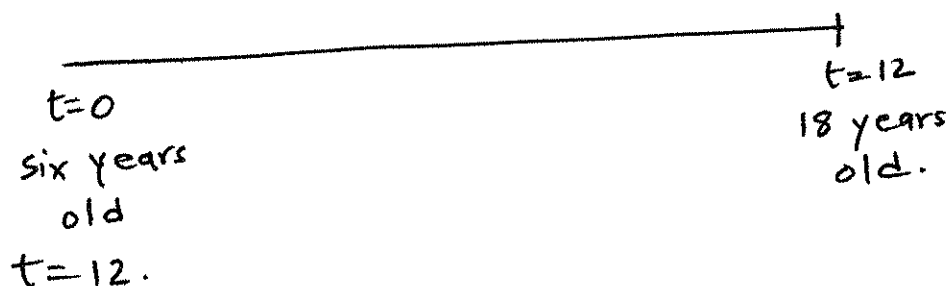
- :- college education
- down payment of a car/house
- Retirement
- etc

✱. Key thing to remember is that one is saving some money every period. This is different from compound interest where the money is put in once as a lumpsum amount and it gathers interest.

3. College Education

\$ 100 Every month

$$\therefore R = 100, m = 12 \quad r = 6\% = 0.06$$



$$F = R \left[\frac{(1+i)^n - 1}{i} \right]$$

$$n = m \cdot t = 12 \times 12 = 144$$

$$i = \frac{r}{m} = \frac{0.06}{12} = 0.005$$

$$F = 100 \left[\frac{\left(1 + \frac{0.06}{12}\right)^{144} - 1}{\left(\frac{0.06}{12}\right)} \right] = 21015.01 \$$$

4. Financing for car down payment

$$F = 5000 \quad m = 12, \quad r = 0.04 \quad t = 4 \text{ years} \quad \text{Find } R$$

$$i = \frac{r}{m} = \frac{0.04}{12}, \quad n = m \cdot t = 12 \times 4 = 48$$

$$F = R \left[\frac{(1+i)^n - 1}{i} \right] = R \left[\frac{\left(1 + \frac{0.04}{12}\right)^{48} - 1}{\left(\frac{0.04}{12}\right)} \right] = 5000$$

$$R = 96.23 \$/\text{month} \text{ have to be saved.}$$

5. Retirement Account.

2000\$ every year.

$$\begin{aligned}
 m &= 1. & r &= 0.09 & n &= m \cdot t \\
 R &= 2000 & t &= 40 & &= 1 \cdot 40 = 40.
 \end{aligned}$$

$$\begin{aligned}
 F &= R \cdot \frac{[(1+i)^n - 1]}{i} \\
 &= 2000 \cdot \frac{[(1+i)^{40} - 1]}{\left(\frac{0.09}{12}\right)} = 675764.89 \$.
 \end{aligned}$$

How much interest is earned.

So each year for 40 years the individual put 2000\$

$\therefore$ Total money he put over 40 years

$$= 2000 \times 40 = 80,000.$$

At the end of 40 years he has accumulated

$$675,764.89 \$.$$

$$\begin{aligned}
 \therefore \text{Total interest earned} &= 675,764.89 - 80,000 \\
 &= 595764.89
 \end{aligned}$$

6. College Education

4.

A) $F = 200,000$

$t = 18 \text{ years}$

$m = 12$

$n = m \cdot t = 18 \times 12 = 216$

$i = \frac{0.09}{12}$

$R = ?$

$$F = R \frac{[(1+i)^n - 1]}{i}$$

$$200,000 = R \frac{\left[\left(1 + \frac{0.09}{12} \right)^{216} - 1 \right]}{\left(\frac{0.09}{12} \right)}$$

$\therefore R = 372.89$

B) IF the parents make a payment of 372.89/month of 18 years, total money they would have put in is

$$372.89 \times (18 \times 12) = 80544.17 \$$$

So total interest they earned in 18 years

$$= 200,000 - 80544.17 = 119,455.83 \$$$

C) Equity in 10 years.

So $F = ?$ after 10 years $n = m \times t = 12 \times 10 = 120$

$i = \frac{r}{m} = \frac{0.09}{12}$ $R = 372.89$

$$\therefore F = \frac{\left(1 + \frac{0.09}{12} \right)^{120} - 1}{\left(\frac{0.09}{12} \right)} \cdot 372.89 = 72159.54 \$$$

D. You want to know how much interest was earned in the 10th year. 5

1st We already know how much money ~~was~~ ^{is} in the account at the end of 10 years 72159.54 \$.

So now find out what is the money after 9 years

$$\therefore n = 9 \times 12 \quad i = \frac{0.09}{12} \quad R = 372.89 \$ \quad F = ?$$

$$F = \frac{(1+i)^n - 1}{i} \cdot R = \frac{\left(1 + \frac{0.09}{12}\right)^{108} - 1}{\left(\frac{0.09}{12}\right)} \cdot 372.89$$
$$= 61707.04 \$.$$

So at the end of 9 years the account has 61,707.04 \$. This is the amount of money @ the beginning of 10th year.

$$\text{So total money in 10th} = 72159.54 - 61707.04$$
$$= 10452.5 \$.$$

This portion has the monthly payments the parents made during the 10th year = $12 \times 372.89 = 4474.68 \$$

So total interest earned in 10th year

$$= 10452.5 - 4474.68 = 5977.82 \$.$$