

5.3

①

15 trans
6 - Def
9 - Good

$$C(15, 2) = \frac{15 \cdot 14}{2} = 105$$

$$C(6, 2) = 15$$

$$p(2 \text{ def trans}) = \frac{C(6, 2)}{C(15, 2)} = \frac{15}{105} = \frac{1}{7}$$

$$C(n, r) = \frac{n!}{r! (n-r)!}$$

$$n = 15$$

$$r = 2$$

$$= \frac{15!}{2! (13)!} = \frac{15 \cdot 14 \cdot \cancel{13!}}{2! \cdot \cancel{13!}}$$

$$= \frac{15 \cdot 14}{2} = 105$$

5 transitors

2 - Bad

3 - Good

$$C(15, 5) = 3003$$

$$C(9, 3) = 84$$

$$C(6, 2) = 15$$

$$\frac{C(6, 2) \cdot C(9, 3)}{C(15, 5)} = \frac{84 \times 15}{3003} = 0.42$$

(3)

5 balls

$$\begin{array}{l} \text{OR} \quad 3W \text{ and } 2R \\ \quad 4W \text{ and } 1R \end{array} + \begin{array}{l} C(4,3) \cdot C(6,2) = 60 \\ C(4,4) \cdot C(6,1) + \frac{6}{66} \end{array}$$

$$=$$

$$P(E) = \frac{66}{252}$$

(10) Standard deck of 52 cards

We are drawing 2 cards

$$\text{Sample} = C(52, 2) = 1326.$$

space

$$\text{Same suit} = C(13, 2) = 78.$$

$$P(E) = \frac{78}{1326} = \frac{1}{17}$$

$$C(52, 2) = 1326$$

$$C(40, 2) = 780$$

$$\frac{780}{1326} = \frac{10}{17}$$

(2)

Toss a coin six times

3 - Heads

$$\left. \begin{matrix} H \\ T \end{matrix} \right\} 2^6$$

$$C(6, 3) = 20.$$

$$P(\text{Head exactly 3 times}) = \frac{20}{2^6} = \frac{20}{64}$$

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(2) Urn has 10 balls.

4 white 6 red

$$C(10, 5) = 252$$

$$3 \text{ W} \quad C(4, 3) = 4$$

$$2 \text{ R} \quad C(6, 2) = 15$$

$$P(3W \& 2R) = \frac{C(4, 3) \cdot C(6, 2)}{C(10, 5)} = \frac{4 \times 15}{252} = \frac{5}{21}$$

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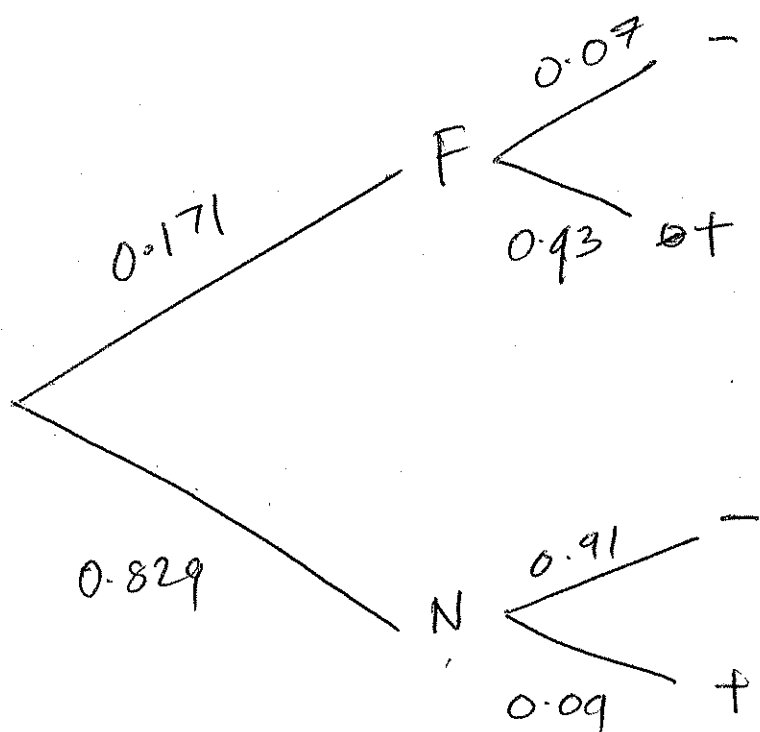
20 $\rightarrow$ 3

10 $\rightarrow$ Above

$C(20, 3) \rightarrow$ sample

$C(10, 3) \rightarrow$

$$\frac{C(10, 3)}{C(20, 3)} = \frac{2}{19}$$



$$P(F|+) = \frac{P(F \cap +)}{P(F) \cdot P(+|F) + P(N) \cdot P(+|N)} = \frac{(0.171)(0.93)}{(0.171)(0.93) + (0.829)(0.09)}$$

5.4

Bernoulli Trials

①

- Flip a coin and see if it is head or Tails
- Test a transistor to see if it is defective or not
- Examine a patient to see if a particular disease is present or not
- Take a free throw in basketball and make the basket or not

FUNDAMENTAL ASSUMPTION FOR BERNOULLI TRIALS

Successive Bernoulli trials are independent from each other

Example 1

S — making the basket

F — not making the basket

A typical sequence could be

SSFSFSFFS

$$\left(\frac{2}{3}\right)\left(\frac{2}{3}\right)\left(\frac{1}{3}\right)\left(\frac{2}{3}\right)\left(\frac{1}{3}\right)\left(\frac{2}{3}\right)\left(\frac{2}{3}\right)\left(\frac{1}{3}\right)\left(\frac{1}{3}\right)\left(\frac{2}{3}\right) = \left(\frac{2}{3}\right)^6 \left(\frac{1}{3}\right)^4$$

Also count the number of ways these 6 success occur
 $C(10, 6)$

(2)

$$C(10, 6) \left(\frac{2}{3}\right)^6 \left(\frac{1}{3}\right)^4 = 0.228$$

Formula.

$$C(n, k) p^k q^{n-k}$$

k = no of successes in n trials

n = no of trials

p = probability of success

q = probability of failure = $1-p$.

Base ball hits

3

8) K in n repeated trials

$$n = 6, k = 4 \quad p = 0.5 \quad q = 1 - p = 0.5$$

$$C(6, 4) \cdot (0.5)^4 \cdot (0.5)^2 = 0.2344.$$

16) $n = 10$

$$\text{head} = \text{success} = p = 0.5$$

$$\text{Tail} = \text{failure} = q = 0.5.$$

$$K = 7, 8, 9, 10$$

$$C(10, 7) \cdot (p)^K (q)^{n-K}$$

$$= C(10, 7) \cdot (0.5)^7 \cdot (0.5)^3 =$$

$$+ C(10, 8) (0.5)^8 (0.5)^2$$

$$+ C(10, 9) (0.5)^9 (0.5)^1$$

$$+ C(10, 10) (0.5)^{10} (0.5)^0$$

$$= 0.172.$$

20) $p(E) = 0.6 \quad q = 0.4.$

$$n = 7 \quad k = 3$$

$$C(7, 3) \cdot (0.6)^3 \cdot (0.4)^4 = 0.1935$$

S - Success of making the free throw in Basket
F = Failure

(1a)

$$(6/10) \quad (4/10)$$

$$p = 6/10 \quad q = 4/10 = 1 - p$$

S S F S F S S F F S

$$\left(\frac{6}{10}\right)\left(\frac{6}{10}\right)\left(\frac{4}{10}\right)\left(\frac{6}{10}\right)\left(\frac{4}{10}\right)\left(\frac{6}{10}\right)\left(\frac{6}{10}\right)\left(\frac{4}{10}\right)\left(\frac{4}{10}\right)\left(\frac{6}{10}\right)$$

$$= \left(\frac{6}{10}\right)^6 \cdot \left(\frac{4}{10}\right)^4$$

$$P(\text{Six go in exactly}) = C(10, 6) \cdot \left(\frac{6}{10}\right)^6 \cdot \left(\frac{4}{10}\right)^4$$

$$= C(n, k) \cdot (p)^k \cdot (q)^{n-k}$$

$$n=6 \quad k=4 \quad p=0.5 \quad q=0.5$$

$$\begin{aligned} P(\text{exactly } 4) &= C(n, k) \cdot (p)^k \cdot (q)^{n-k} \\ &= C(6, 4) (0.5)^4 \cdot (0.5)^2 \\ &= 0.2344 \end{aligned}$$

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$$\begin{aligned} \text{Head} - \text{Success} &= 0.5 = p \\ \text{Tail} - \text{Failure} &= 0.5 = q \end{aligned}$$

$$n=10 \quad p=0.5 \quad q=0.5$$

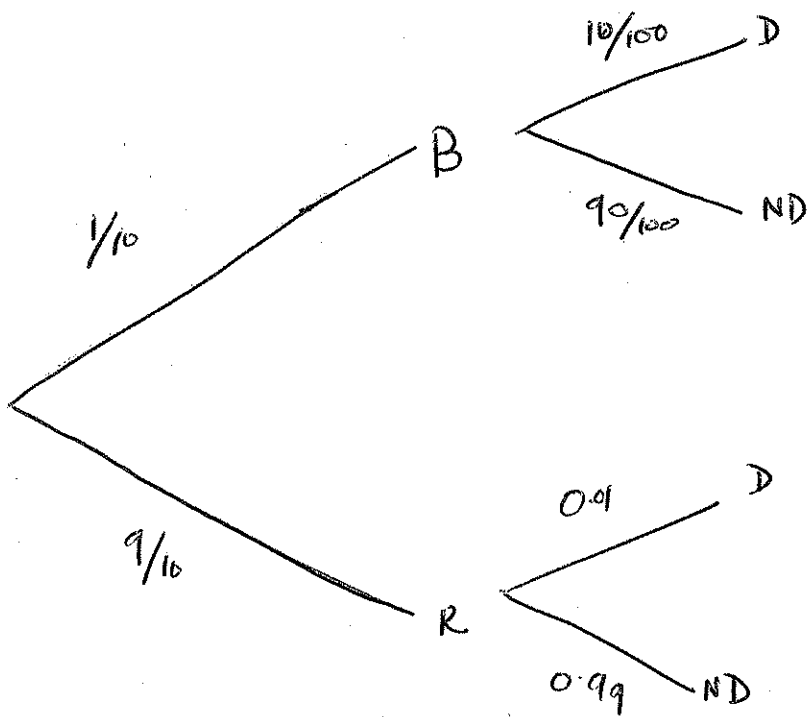
$$\begin{aligned} &C(10, 7) (0.5)^7 \cdot (0.5)^3 \\ &+ C(10, 8) (0.5)^8 \cdot (0.5)^2 \\ &+ C(10, 9) (0.5)^9 \cdot (0.5)^1 \\ &+ C(10, 10) (0.5)^{10} \cdot (0.5)^0 = 0.172 \end{aligned}$$

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$$\begin{aligned} p(E) &= 0.6 \quad q = 0.4 \\ n &= 7 \quad k = 3 \end{aligned}$$

$$C(7, 3) (0.6)^3 (0.4)^4 = 0.1935$$

(1a)



$$\begin{aligned}
 P(B|DND) &= \frac{P(B \cap DND)}{P(B) \cdot P(DND|B) + P(R) \cdot P(DND|R)} \\
 &= \frac{0.1 [(0.1)(0.9)]}{(0.1)(0.1)(0.9) + (0.9)(0.1)(0.99)} = 0.5025
 \end{aligned}$$

Also can use combination.

$$\begin{aligned}
 &= 0.1 \frac{C(10,1) \cdot C(90,1)}{C(100,2)} \\
 &\quad \frac{0.1 \left(\frac{C(10,1) \cdot C(90,1)}{C(100,2)} \right) + 0.9 \left(\frac{C(1,1) \cdot C(99,1)}{C(100,2)} \right)}{C(100,2)} \\
 &= 0.5025.
 \end{aligned}$$

(30)

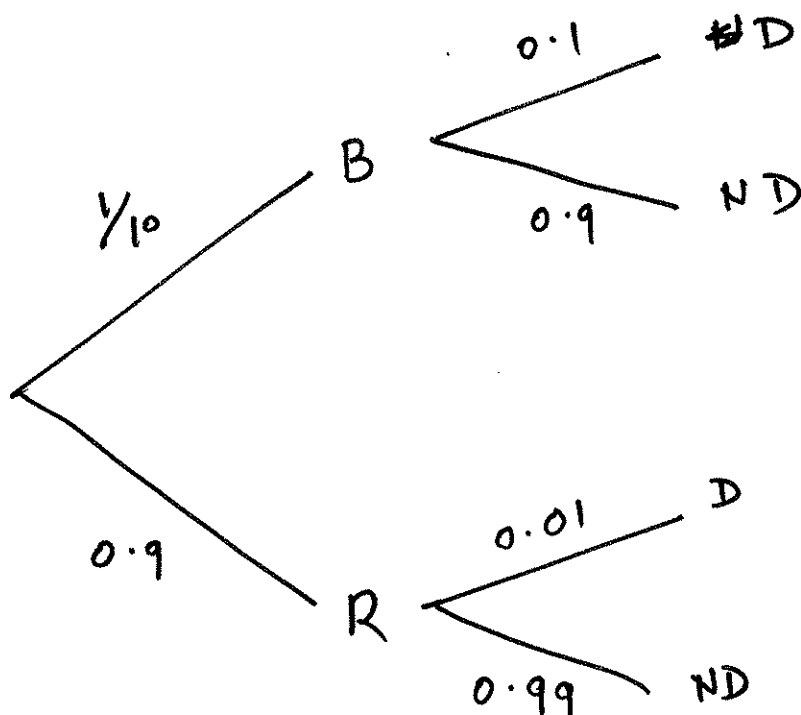
10 boxes 1 box $\rightarrow$ 10 $\rightarrow$ D-defective

$$D = \frac{10}{100} = 0.1$$

$$ND = 0.9$$

9 boxes $\rightarrow$ 1 $\rightarrow$ D

$$\therefore D = 1/100 \quad ND = 99/100$$



$$\begin{aligned}
 P(B|DND) &= \frac{P(B \cap DND)}{P(B) \cdot P(DND|B) + P(R) \cdot P(D \cdot ND|R)} \\
 &= \frac{(0.1)(0.1)(0.9)}{(0.1)(0.1)(0.9) + (0.9)(0.01)(0.99)} = 0.5025
 \end{aligned}$$

(36)

$$p = 1/20$$

$$q = 19/20$$

$$n = 12$$

$$k = 4$$

$$C(12, 4) = \left(\frac{1}{20}\right)^4 \left(\frac{19}{20}\right)^8$$

$$= 0.00205.$$

(4)

(38)

$$q = 1/5$$

$$p = 4/5$$

$$n = 20$$

$$k = 1 \text{ or } 2$$

$$C(20, 1) \left(\frac{4}{5}\right)^1 \left(\frac{1}{5}\right)^{19} \\ + C(20, 2) \left(\frac{4}{5}\right)^2 \left(\frac{1}{5}\right)^{18}$$

$$\approx \underline{3.27 \times 10^{-11}}$$

4a

$$F = 1/5 = q$$

$$S = 4/5 = p$$

$$n = 20$$

At most 2

meaning ① OR ②

$$K=1 \text{ or } K=2$$

$$C(20,1) \cdot \left(\frac{4}{5}\right)^1 \left(\frac{1}{5}\right)^{19} + C(20,2) \left(\frac{4}{5}\right)^2 \left(\frac{1}{5}\right)^{18}$$
$$= 3.27 \times 10^{-11}$$