

Chapter 5 Counting & Probability. ①

KEY

5.1 The multiplication Principle &

Permutations

Factorials

Number of Permutations of n objects

Number of Permutations of ' r ' objects
taken from a set of size N

5.2

Combinations

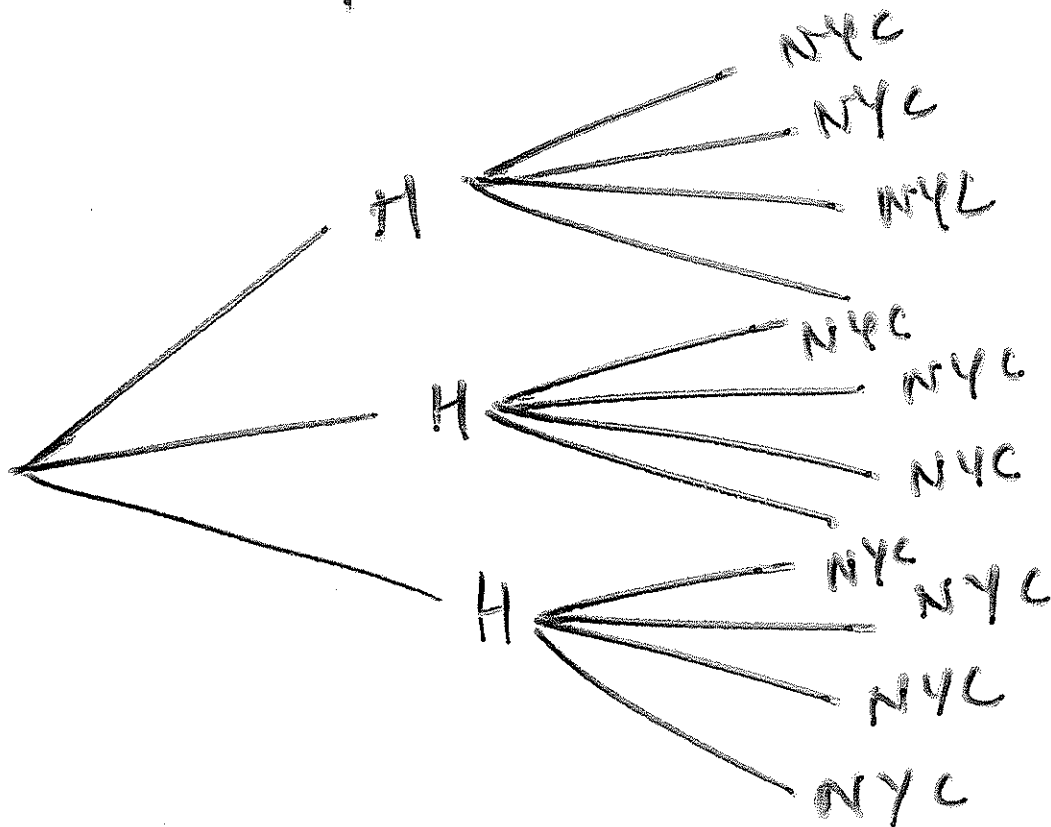
A combination of ' r ' objects taken from a
set of size ' N '

Problems & Applications

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LANCE NYE C-3007

Multiplication Principle



3 ways to go to Hartford

4 ways to go to NYC from Hartford

So $3 \times 4 = 12 \Rightarrow$ to go to NYC from
stores.

Factorials

$$n! = n(n-1)(n-2)\dots 3 \cdot 2 \cdot 1$$

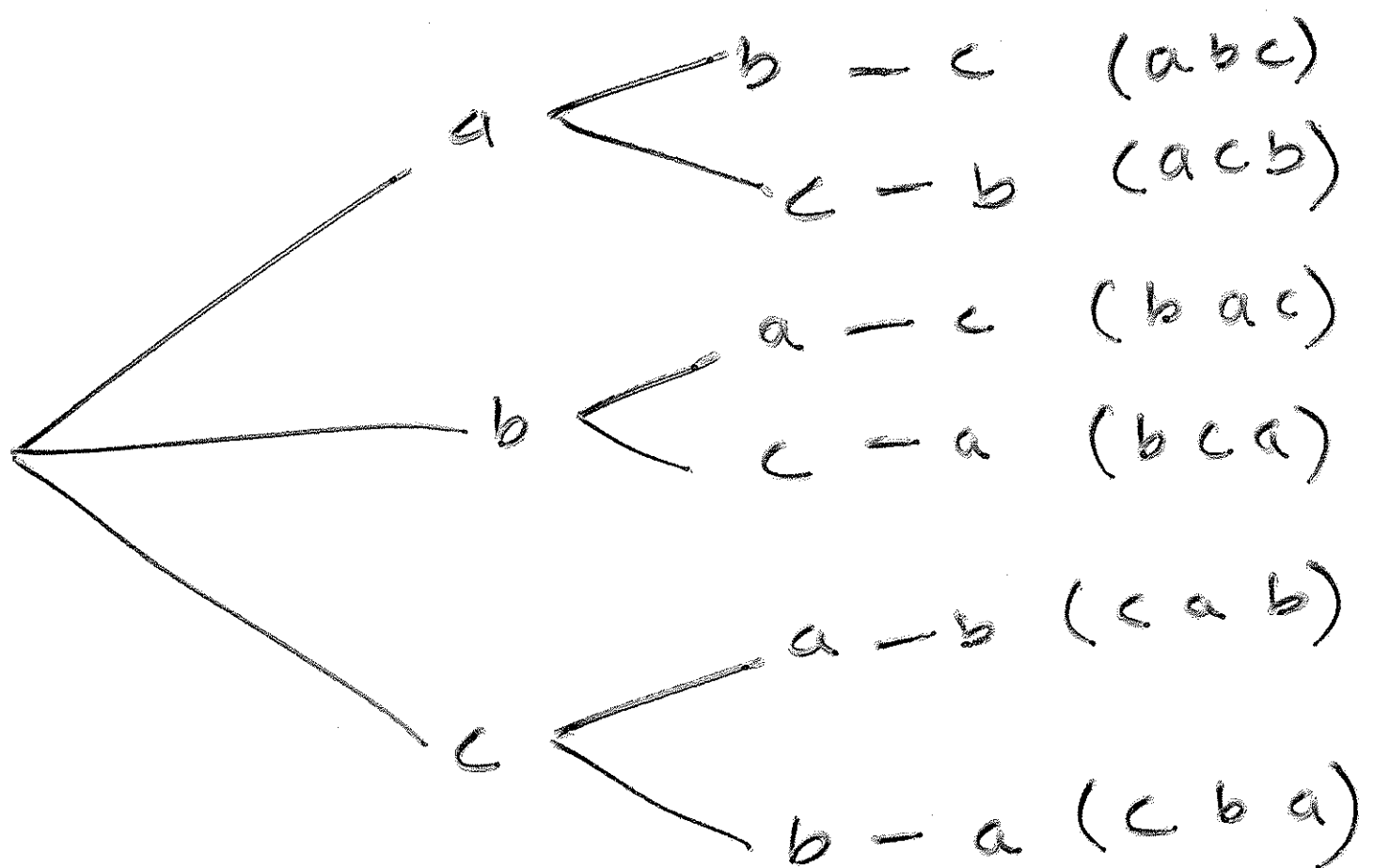
$$4! = 4 \cdot (4-1) (4-2) (4-3) \\ = 4 \cdot 3 \cdot 2 \cdot 1 = 24$$

$$6! = 6 \cdot 5 \cdot \overset{4}{\underset{\uparrow}{3}} \cdot 2 \cdot 1 = 720$$

Permutation

{ a b c }

abc
acb
~~b~~ac
bca
cab
cba



$$3 \cdot 2 \cdot 1 = 6 \text{ ways}$$

a b c

$$3 \cdot 2 \cdot 1 = 6 \text{ ways} = 3!$$

Number of Permutations of n objects
 $= P(n, n) = n(n-1)(n-2) \dots 3 \cdot 2 \cdot 1$
 $= n!$

EX 10

$$\begin{array}{cccccc} L & L & L & D & D & D \\ 26 & 25 & 24 & 10 & 9 & 8 \end{array} = 11,232,000$$

$$\begin{array}{cc} 26 & 10 \\ 3 & 3 \end{array}$$

$$P(26,3) \cdot P(10,3) = 11,232,000$$

$$P(n,r) = \frac{n!}{(n-r)!}$$

$$P(10,3) = \frac{10!}{(10-3)!} = \frac{10!}{7!}$$

A combination of r distinct objects taken from a set of size N is a selection of r of the object (without concern for order)

Problems

5.1 & 5.2.

①

$$\textcircled{2} \quad P(5,2) = \frac{5!}{(5-2)!} = \frac{5!}{3!} = \frac{5 \times 4 \cdot 3!}{3!} = 20.$$

⑩ 52 cards $\rightarrow$ can be picked in 52 ways
Coin is flipped two possible outcomes

$$\therefore 52 \times \underset{\substack{\uparrow \\ \text{each time} \\ \text{coin is tossed}}}{2} \times 2 = 208$$

⑫ 4 styles of home

Each home has 3 types of garages
4 types of decks
5 styles of carpeting

Multiplication principle

$$4 \times 3 \times 4 \times 5 = 240$$

(26) 5 - women 4 - men

(2)

$$\begin{array}{cccccccc} W & W & M & W & M & W & M & W & M \\ 5 & 4 & 3 & 3 & 2 & 2 & 1 & 1 & \end{array} = 2880.$$

(32) OL — 7
OB — 4.

OB OB OL OL OL OL OL OL OL OB OB

$P(4,2)$ $P(7,7)$ $P(2,2)$

$$\frac{4!}{2!} \times 7! \times 2! = 120,960 \text{ ways.}$$

(36) This is a problem of permutations of 4 horses chosen from a group of 10 horses
(Notice that 4 positions arranged/assigned ^{are} ~~are~~ distinct from one another.)

Hence the number of ways of selection

$$\text{is } P(10,4) = 10 \times 9 \times 8 \times 7 = 5040.$$

$$P(5,2) =$$

2a

$$P(n,r) = \frac{n!}{(n-r)!} = \frac{5!}{(5-2)!} = \frac{5!}{3!} = 20$$

$$5! = \frac{5 \times 4 \times \cancel{3} \times \cancel{2} \times \cancel{1}}{\cancel{3} \times \cancel{2} \times \cancel{1}} = \underline{\underline{20}}$$

5 - W

4 - M

5!
4!

$$\begin{array}{cccccccc} W & W & M & W & M & W & M & W & M \\ 5 & 4 & 4 & 3 & 3 & 2 & 2 & 1 & 1 \end{array} = 2880$$

OL - 7

OB = 4

$$OB \ OB \ (OL \ OL \ OL \ OL \ OL \ OL \ OL \ OL) \ OB \ OB$$

$$P(4,2) \quad P(7,7) \quad P(2,2)$$

(3)

(40) First race runs 8 horses

Pick two. $P(8, 2) = 8 \times 7 = 56$

Second race runs 10 horses.

Pick two $P(10, 2) = 10 \times 9 = 90$

Multiplication principle

$$56 \times 90 = 5040.$$

5.2

$$(10) \quad C(n, 0) = \frac{n!}{0! \cdot n!} = 1.$$

(16)

40 captains.

5 are to be promoted

$$C(40, 5) = \frac{40!}{5! \cdot 35!} = 658,008.$$

(26)

12 US senators 7 - D 5 - R

53 - D 47 - R.

 $C(53, 7) \rightarrow$ Democrats chosen $C(47, 5) \rightarrow$ RepublicansTogether $C(53, 7) \cdot C(47, 5)$

(36) Coin is flipped 8 times

In how many ways 6 head occurs

At least '6' heads = number of exactly k heads occur. $k = 6, 7, 8$.

$$N = C(8, 6) + C(8, 7) + C(8, 8) = 28 + 8 + 1 = 37.$$

(38) Winning 7 of 12 is same as losing 5 of 12

$$C(12, 7) = C(12, 5) = \frac{12!}{7! \times 5!} = 792.$$

(44) 8 boys 4 girls.

First select 2 boys & 2 girls

$$N_1 = C(8, 2) \times C(4, 2) = 28 \times 6 = 168$$

There are two ways of making different pairs for selected boys and girls so total no of ways a selection is made

$$N = N_1 \times N_2 = 168 \times 2 = \underline{\underline{336}}$$