

4.3 Sample Spaces & Events (8)

Experiment is an activity that has an observable result

Outcome is the result of experiment

Sample space of an experiment is a set of all possible outcomes of experiment

Each repetition of an experiment is called trial

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$$S = \{ (1,1) (1,2) (1,3) (1,4) (1,5) (1,6) \\ (2,1) (2,2) (2,3) (2,4) (2,5) (2,6) \\ (3,1) \\ (4,1) \\ (5,1) \\ (6,1) (6,2) (6,3) (6,4) (6,5) (6,6) \}$$

EXAMPLE 1 Determining the Sample Space An experiment consists of noting whether the price of the stock of the Ford Corporation rose, fell, or remained unchanged on the most recent day of trading. What is the sample space for this experiment?

Solution There are three possible outcomes depending on whether the price rose, fell, or remained unchanged. Thus the sample space S is

$$S = \{\text{rose, fell, unchanged}\}$$



EXAMPLE 2 Determining the Sample Space Two dice, identical except that one is green and the other is red, are tossed and the number of dots on the top face of each is observed. What is the sample space for this experiment?

Solution Each die can take on its six different values with the other die also taking on all of its six different values. We can express the outcomes as order pairs. For example, (2, 3) will mean 2 dots on the top face of the green die and 3 dots on the top face of the red die. The sample space S is below. A more colorful version is shown in Figure 4.15.

$$S = \{(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), \\ (2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6), \\ (3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6), \\ (4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6), \\ (5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6), \\ (6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)\}$$



If the experiment of tossing 2 dice consists of just observing the total number of dots on the top faces of the two dice, then the sample space would be

$$S = \{2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$$

In short, the sample space depends on the precise statement of the experiment.

EXAMPLE 3 Determining the Sample Space A coin is flipped twice to observe whether heads or tails shows; order is important. What is the sample space for this experiment?

Solution The sample space S consists of the 4 outcomes

$$S = \{(H, H), (H, T), (T, H), (T, T)\}$$



✧ Tree Diagrams

In Example 3 we completed a task (flipped a coin) and then completed another task (flipped the coin again). In these cases, the experiment can be diagrammed with a tree. The tree diagram for Example 3 is shown in Figure 4.16. We see we have a first set of branches representing the first flip of the coin. From there we flip the coin again and have a second

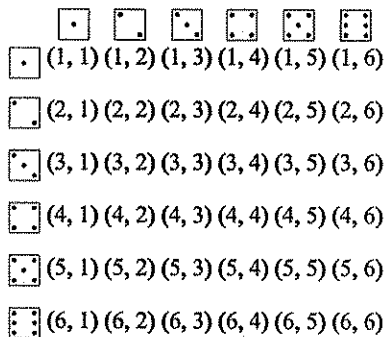


Figure 4.15

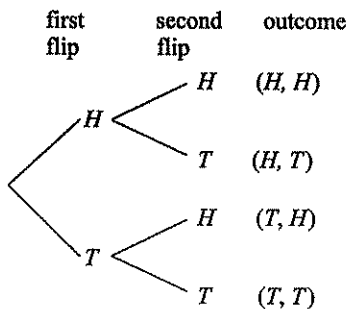
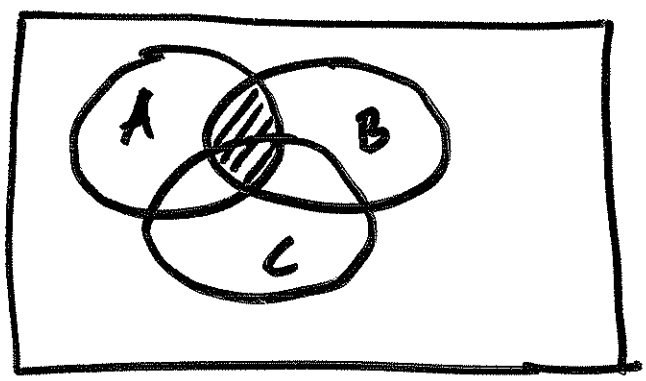


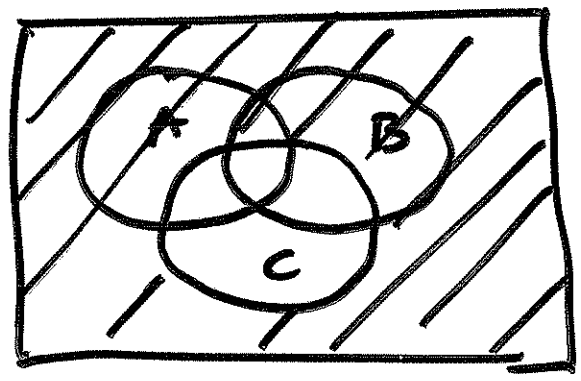
Figure 4.16

①

$$A \cap B \cap C^c$$

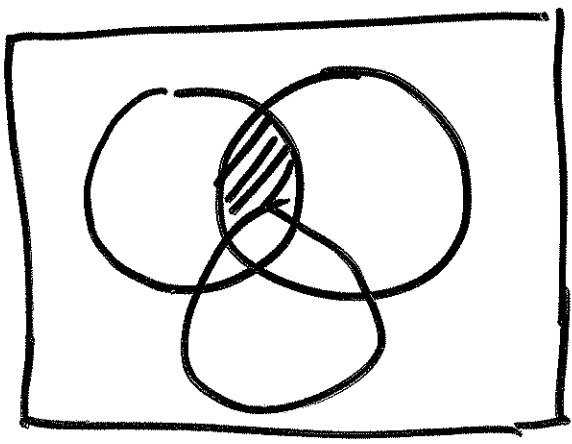


$$A \cap B$$

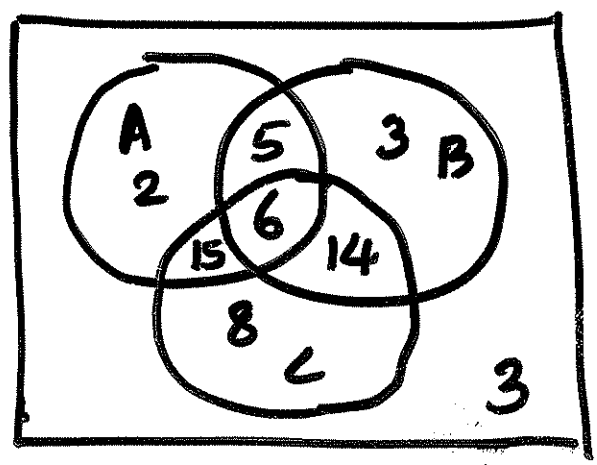


$$(A \cup B \cup C)^c$$

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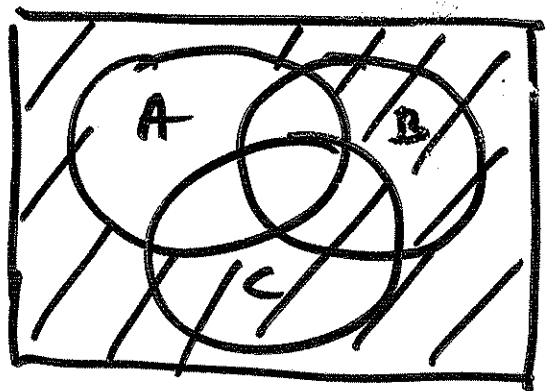


$$A \cap B \cap C^c = 5$$

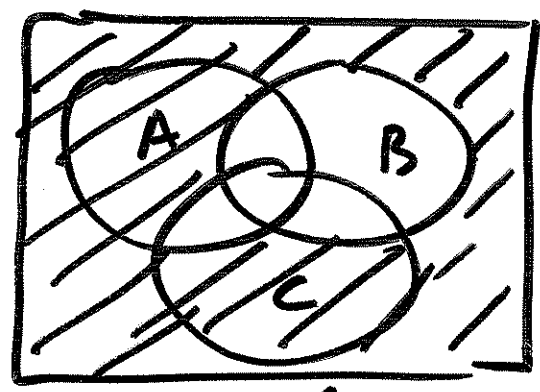


$$n(A \cup B \cup C)^c = 3$$

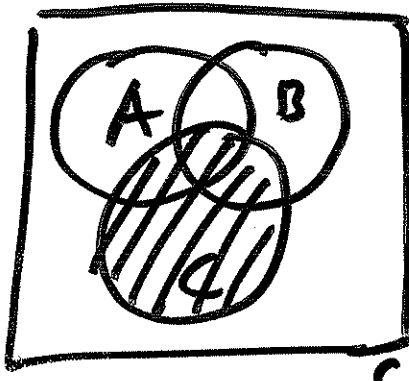
$$n(A^c \cap B^c \cap C) = 8$$



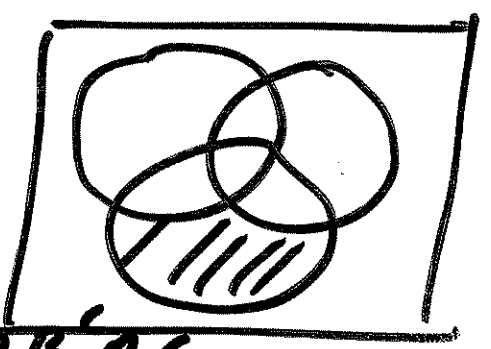
$$A^c$$



$$B^c$$



$$A^c \cap B^c \cap C$$

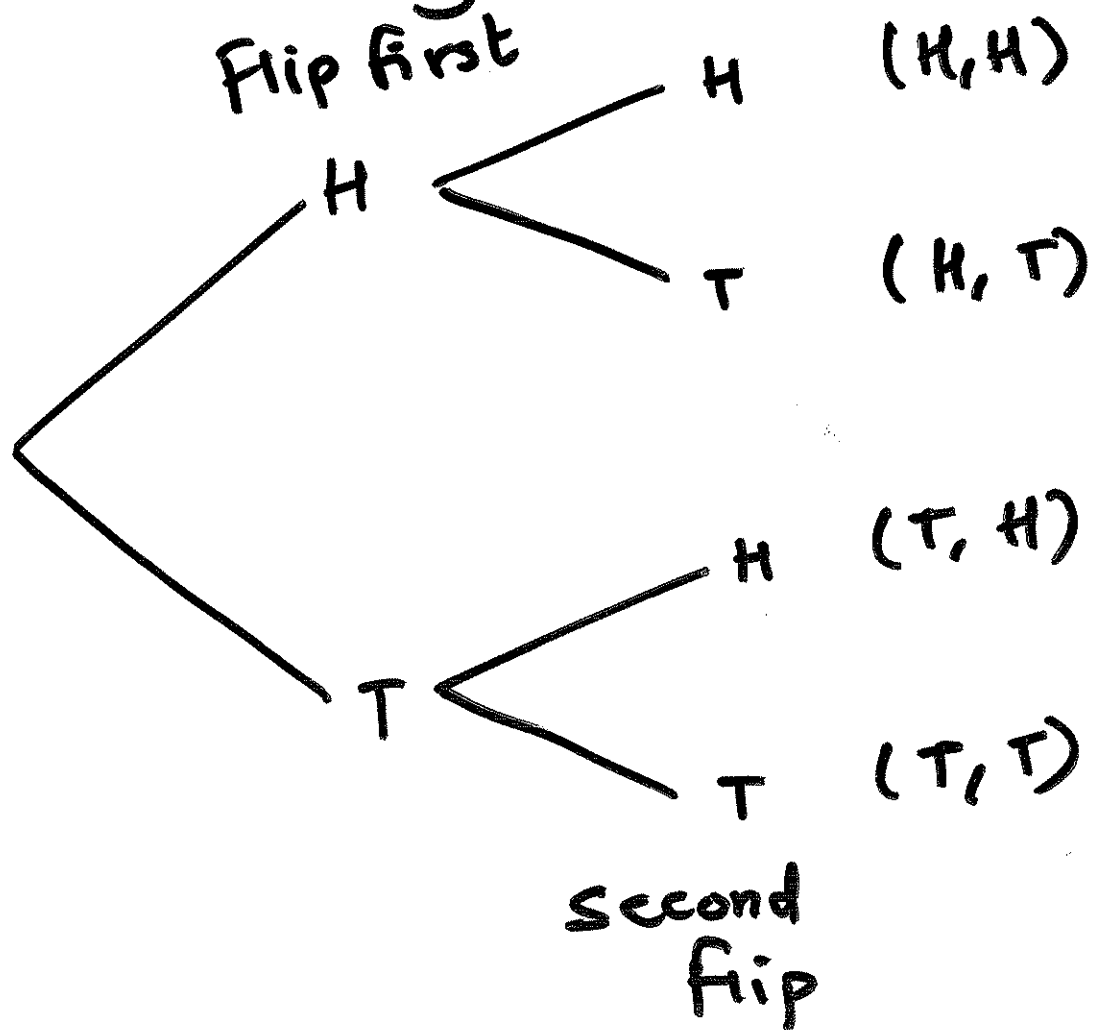


Toss a coin twice

②

$$S = \{(H, H), (T, T), (H, T), (T, H)\}$$

Tree diagram



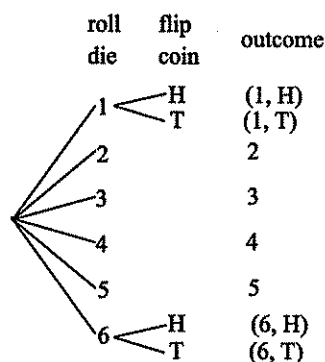


Figure 4.17

set of branches. Then trace along each branch to find the outcomes of the experiment. If the coin is tossed a third time, there will be eight outcomes.

EXAMPLE 4 Determining the Sample Space A die is rolled. If the die shows a 1 or a 6, a coin is tossed. What is the sample space for this experiment?

Solution Figure 4.17 shows the possibilities. We then have

$$S = \{(1, H), (1, T), 2, 3, 4, 5, (6, H), (6, T)\}$$

Events

We start this subsection with the following definition of an event.

Events and Elementary Events

Given a sample space S for an experiment, an event is any subset E of S . An elementary (or simple) event is an event with a single outcome.

EXAMPLE 5 Finding Events Using the sample space from Example 3 find the events: “At least one head comes up” and “Exactly two tails come up.” Are either events elementary events?

Solution “At least one head comes up” = $\{(H, H), (H, T), (T, H)\}$

“Exactly two tails come up” = $\{(T, T)\}$

The second event, “Exactly two tails come up” has only one outcome and so it is an elementary event.

We can use our set language for union, intersection, and complement to describe events.

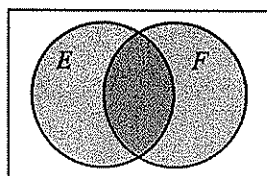


Figure 4.18

Union of Two Events

If E and F are two events, then $E \cup F$ is the union of the two events and consists of the set of outcomes that are in E or F .

Thus the event $E \cup F$ is the event that “ E or F occurs.” Refer to Figure 4.18 where the event $E \cup F$ is the shaded region on the Venn diagram.

Intersection of Two Events

If E and F are two events, then $E \cap F$ is the intersection of the two events and consists of the set of outcomes that are in both E and F .

Thus the event $E \cap F$ is the event that “ E and F both occur.” Refer to Figure 4.18 where the event $E \cap F$ is the region where E and F overlap.

Complement of an Event

If E is an event, then E^c is the complement of E and consists of the set of outcomes that are not in E .

Thus the event E^c is the event that “ E does not occur.”

EXAMPLE 6 Determining Union, Intersection, and Complement Consider the sample space given in Example 2. Let E consist of those outcomes for which the number of dots on the top faces of both dice is 2 or 4. Let F be the event that the sum of the number of dots on the top faces of the two dice is 6. Let G be the event that the sum of the number of dots on the top faces of the two dice is less than 11.

- List the elements of E and F .
- Find $E \cup F$.
- Find $E \cap F$.
- Find G^c .

Solution

- $E = \{(2, 2), (2, 4), (4, 2), (4, 4)\}$ and $F = \{(1, 5), (2, 4), (3, 3), (4, 2), (5, 1)\}$
- $E \cup F = \{(2, 2), (2, 4), (4, 2), (4, 4), (1, 5), (3, 3), (5, 1)\}$
- $E \cap F = \{(2, 4), (4, 2)\}$
- $G^c = \{(5, 6), (6, 5), (6, 6)\}$ ♦

If S is a sample space, $\emptyset \subseteq S$, and thus $\emptyset$ is an event. We call the event $\emptyset$ the impossible event since the event $\emptyset$ means that no outcome has occurred, whereas, in any experiment some outcome *must* occur.

The Impossible Event

The empty set, $\emptyset$, is called the impossible event.

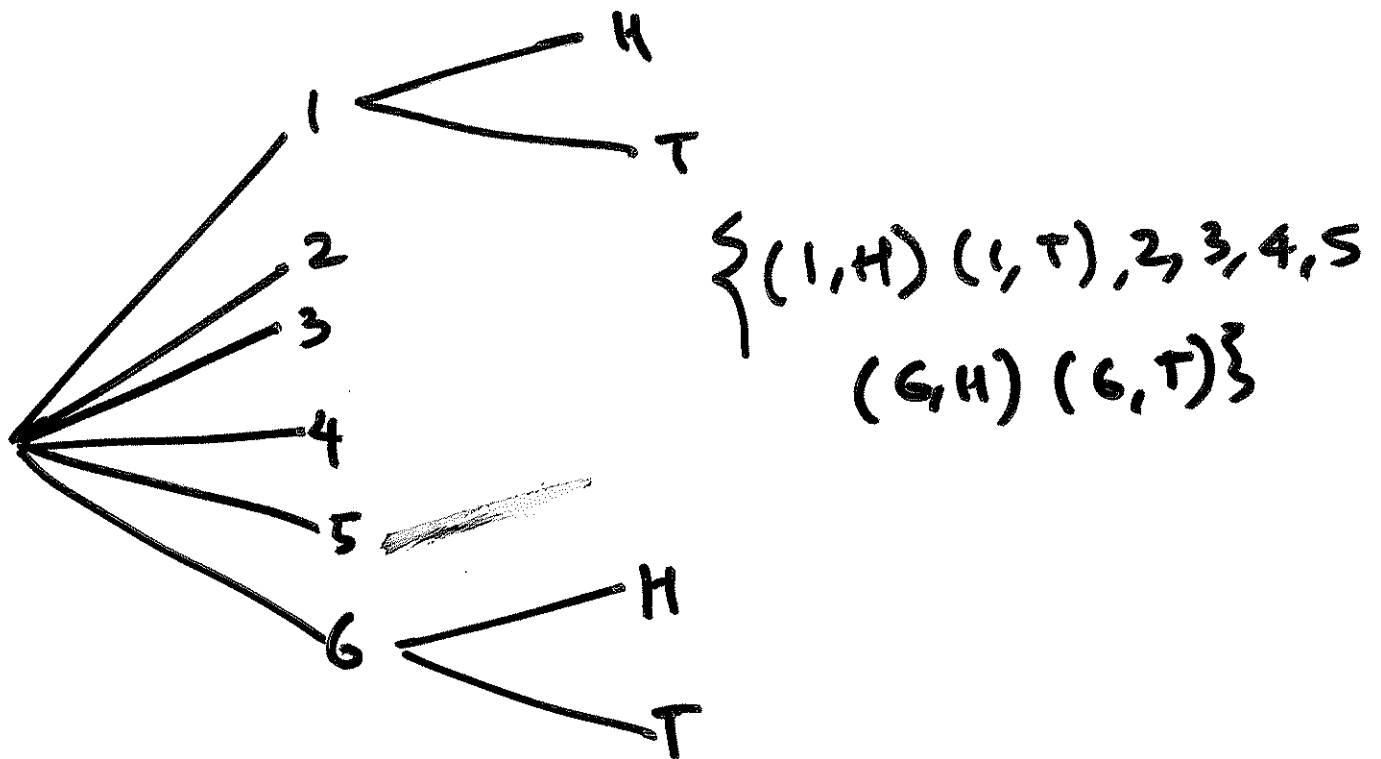
For example, if H is the event that a head shows on flipping a coin and T is the event that a tail shows, then $H \cap T = \emptyset$. The event $H \cap T$ means that both heads and tails shows, which is impossible.

Since $S \subseteq S$, S is itself an event. We call S the certainty event since any outcome of the experiment must be in S . For example, if a fair coin is flipped, the event $H \cup T$ is certain since a head or tail must occur.

The Certainty Event

Let S be a sample space. The event S is called the certainty event.

For Rolling a Dice & if I get ③
 1 or 6 $\rightarrow$ I Flip a coin



$$E = \{(2,2), (2,4), (4,2), (4,4)\}$$

$$F = \{(1,5), (2,4), (3,3), (4,2), (5,1)\}$$

$$E \cap F = \{(2,4), (4,2)\}$$

$$E \cup F = \{(2,2), (2,4), (4,2), (4,4), (1,5), (5,1), (3,3)\}$$

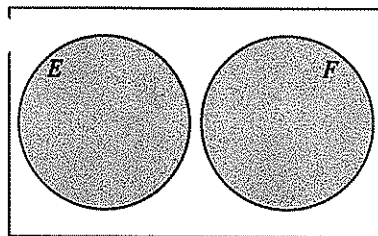


Figure 4.19

We also have the following definition for mutually exclusive events. See Figure 4.19.

Mutually Exclusive Events

Two events E and F are said to be mutually exclusive if the sets are disjoint. That is,

$$E \cap F = \emptyset$$

Standard Deck of 52 Playing Cards

A standard deck of 52 playing cards has four 13-card suits: clubs ♣, diamonds ♦, hearts ♥, and spades ♠. The diamonds and hearts are red, while the clubs and spades are black. Each 13-card suit contains cards numbered from 2 to 10, a jack, a queen, a king, and an ace. The jack, queen, king, and ace can be considered respectively as number 11, 12, 13, and 14. In poker the ace can be either a 14 or a 1. See Figure 4.20.

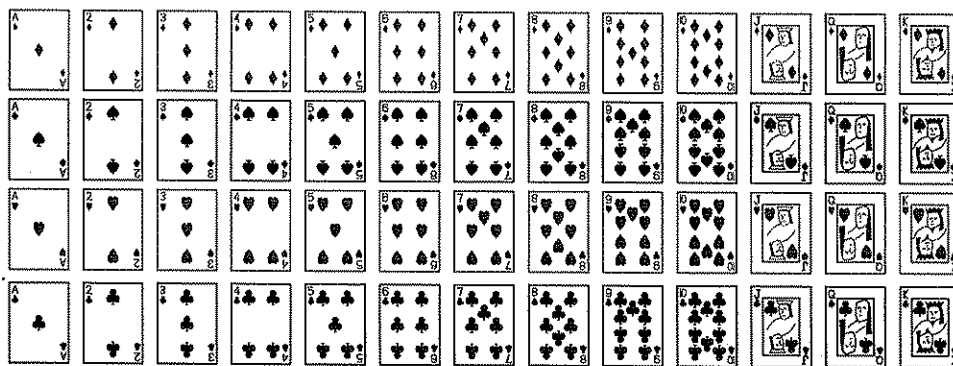


Figure 4.20

EXAMPLE 7 Determining if Sets Are Mutually Exclusive Let a card be chosen from a standard deck of 52 cards. Let E be the event consisting of drawing a 3. Let F be the event of drawing a heart. Let G be the event of drawing a Jack. Are E and F mutually exclusive? Are E and G ?

Solution Since $E \cap F$ is the event that the card is a 3 and is a heart,

$$E \cap F = \{3\heartsuit\} \neq \emptyset$$

and so these events are not mutually exclusive. The event $E \cap G$ is the event that the card is a 3 and a jack, so

$$E \cap G = \emptyset$$

therefore E and G are mutually exclusive. ♦

✧ Continuous Sample Spaces

In all the previous examples we were able to list each outcome in the sample space, even if the list is rather long. But consider the outcomes of an experiment where the time spent running a race is measured. Depending on how the time is measured, an outcome could be 36 seconds or 36.0032 seconds. The values of the outcomes are not restricted to whole numbers and so the sample space must be described, rather than listed. In the case of the race we could say $S = \{t | t \geq 0, t \text{ in seconds}\}$. Then the event E that a person takes less than 35 seconds to run the race would be written $E = \{t | t < 35 \text{ seconds}\}$.

EXAMPLE 8 Weighing Oranges At a farmer's market there is a display of fresh oranges. The oranges are carefully weighed. What is a sample space for this experiment? Describe the event that an orange weighs 100 grams or more. Describe the event that an orange weighs between 200 and 250 grams.

Solution Since the weight of the orange can be any positive number,

$$S = \{w | w > 0, w \text{ in grams}\}$$

Note that $w = 0$ is not included as if the weight was zero there would be no orange! The event that an orange weighs 100 grams or more is

$$E = \{w | w \geq 100, w \text{ in grams}\}$$

Here note that we use $\geq$, not $>$ as the value of exactly 100 grams needs to be included. The event that the orange weighs between 200 and 250 grams is

$$F = \{w | 200 < w < 250, w \text{ in grams}\}$$

where strict inequalities are used as the weight is between those values. ✧

Self-Help Exercises 4.3

1. Two tetrahedrons (4 sided), each with equal sides numbered from 1 to 4, are identical except that one is red and the other green. If the two tetrahedrons are tossed and the number on the bottom face of each is observed, what is the sample space for this experiment?
 - a. List the elements of E and F .
 - b. Find $E \cap F$.
 - c. Find $E \cup F$.
 - d. Find G^c .
2. Consider the sample space given in the previous exercise. Let E consist of those outcomes for which both (tetrahedron) dice show an odd number. Let F be the event that the sum of the two numbers on these dice is 5. Let G be the event that the sum of the two numbers is less than 7.
 - a. the baby is longer than 22 inches.
 - b. the baby is 20 inches or shorter.
 - c. the baby is between 19.5 and 21 inches long.
3. A hospital carefully measures the length of every baby born. What is a sample space for this experiment? Describe the events
 - a. the baby is longer than 22 inches.
 - b. the baby is 20 inches or shorter.
 - c. the baby is between 19.5 and 21 inches long.

4.3 Sample Spaces and Events

Many people have a good idea of the basics of probability. That is, if a fair coin is flipped, you have an equal chance of a head or a tail showing. However, as we proceed to study more advanced concepts in probability we need some formal definitions that will both agree with our intuitive understanding of probability and allow us to go deeper into topics such as conditional probability. This will tie closely to work we have done learning about sets.

✧ The Language of Probability

We begin the preliminaries by stating some definitions. It is very important to have a clear and precise language to discuss probability so pay close attention to the exact meanings of the terms below.

Experiments and Outcomes

An experiment is an activity that has observable results.

An outcome is the result of the experiment.

The following are some examples of experiments. Flip a coin and observe whether it falls “heads” or “tails.” Throw a die (a small cube marked on each face with from one to six dots¹) and observe the number of dots on the top face. Select a transistor from a bin and observe whether or not it is defective.

The following are some additional terms that are needed.

Sample Spaces and Trials

A sample space of an experiment is the set of all possible outcomes of the experiment. Each repetition of an experiment is called a trial.

For the experiment of throwing a die and observing the number of dots on the top face the sample space is the set

$$S = \{1, 2, 3, 4, 5, 6\}$$

In the experiment of flipping a coin and observing whether it falls heads or tails, the sample space is $S = \{\text{heads}, \text{tails}\}$ or simply $S = \{H, T\}$.

¹There are also four-sided die, eight-sided die, and so on. However, the six-sided die is the most common and six-sided should be assumed when we refer to a die, unless otherwise specified.

④

② 4.3 page 207

$$S = \{a, b, c, d\} = 4.$$

$$\textcircled{6} \quad S = \left\{ \begin{array}{cccc} (1,1) & (1,2) & (1,3) & (1,4) \\ (2,1) & (2,2) & (2,3) & (2,4) \\ (3,1) & (3,2) & (3,3) & (3,4) \\ (4,1) & (4,2) & (4,3) & (4,4) \end{array} \right\}$$

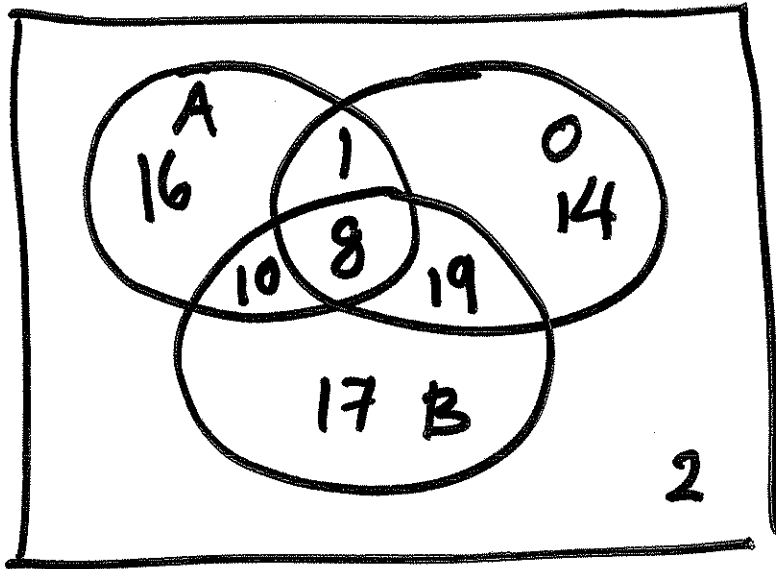
$$E = \{ (1,3), (2,2), (3,1) \}$$
$$= \underline{\underline{3}}$$

⑧ 10 - Balls
1 - white
4 - B
5 - R

$$\left\{ \begin{array}{cc} (W,B) & (R,W) \\ (W,R) & (R,B) \\ (B,W) & (R,R) \\ (B,R) & \\ (B,B) & \end{array} \right\}$$

$$\{ (B,R) (B,B) (R,B) (R,R) \}$$

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✓
1
2
✓
3
✓
4
✓
5
✓
6
7

- 1) 19
- 2) 14
- 3) 35