

# Chapter 3

①

## Linear Programming

### Section 3.1

- Linear
- Objective function
- Constraints
- Feasible Region
- Optimize a problem

### Key

Translate question into a linear programming problem

### 3.2

Solve linear inequalities

Feasible region

✧ Linear Programming Applications

**EXAMPLE 1 Maximizing Profits From Benches** A shop manufactures two styles of aluminum park benches, fancy and plain. The fancy style requires 4 kilograms of aluminum and 6 hours of labor, while the plain style requires 4 kilograms of aluminum and 3 hours of labor. There are 32 kilograms of aluminum and 36 hours of labor available for the day. The fancy style bench earns a profit of \$24 and the plain one \$18. Naturally, the shop's objective is to maximize its profits. Write a system of inequalities that expresses this information, determine the objective function, and write all this as a linear programming problem.

**Solution** Begin by defining the variables. Let  
 $x$  = the number of fancy benches made  
 $y$  = the number of plain benches made  
The information given in the problem is summarized in the table below.

|                       | Fancy | Plain | Available |
|-----------------------|-------|-------|-----------|
| Kilograms of Aluminum | 4     | 4     | 32        |
| Hours of Labor        | 6     | 3     | 36        |
| Profit                | \$24  | \$18  |           |

Since each fancy bench requires 4 kilograms of aluminum, there is a total of  $4x$  kilograms of aluminum for these benches. Since each plain bench requires 4 kilograms of aluminum, there are a total of  $4y$  kilograms of aluminum for these benches. Then  $4x + 4y$  is the total number of kilograms of aluminum needed to make both of the benches, while the total amount available is 32 kilograms. Thus,  $4x + 4y \leq 32$ .

Since each fancy bench requires 6 hours of labor, there are a total of  $6x$  hours of labor needed for these benches. Since each plain bench requires 3 hours of labor, there are a total of  $3y$  hours of labor needed for these benches. Then  $6x + 3y$  is the total number of hours of labor needed to make both types of the benches, while the total labor available is 36 hours. Thus,  $6x + 3y \leq 36$ .

Naturally, we must have  $x \geq 0$  and  $y \geq 0$ , which we refer to as the nonnegativity constraint. We then have the system of inequalities (constraints):

$$\begin{aligned} 4x + 4y &\leq 32 && \text{kilograms of aluminum constraint} \\ 6x + 3y &\leq 36 && \text{hours of labor constraint} \\ x, y &\geq 0 && \text{nonnegativity constraint} \end{aligned}$$

The profit function in dollars is  $P = 24x + 18y$ . Our objective is to maximize this function.

$$\begin{aligned} \text{Maximize} \quad & P = 24x + 18y && \text{objective function} \\ \text{Subject to} \quad & 4x + 4y \leq 32 && \text{kilograms of aluminum} \\ & 6x + 3y \leq 36 && \text{hours of labor} \\ & x, y \geq 0 && \text{nonnegativity} \end{aligned}$$

Notice that we now have several possibilities for the number of benches produced. For example, the shop could make one fancy bench and one plain



(2)

 $X^1 \rightarrow \text{Linear}$ 
 $X^2 \rightarrow \text{Quadratic}$ 
 $X^3 \rightarrow \text{Cubic}$ 

|          | $X$<br>Fancy | $y$<br>Plain | Avail |
|----------|--------------|--------------|-------|
| kg Al    | 4            | 4            | 32    |
| Labor Hr | 6            | 3            | 36    |
| Profit   | 24           | 18           |       |

$$4x + 4y \leq 32$$

$$6x + 3y \leq 36$$

$$P = 24x + 18y$$

$$x \geq 0 ; y \geq 0$$

$$x \geq 4$$

$$x \leq 0$$

$$x = 4$$

|          | D  | S  | Avai |
|----------|----|----|------|
| cutting  | 2  | 4  | 80   |
| Assembly | 4  | 2  | 50   |
| paint    | 2  | 2  | 50   |
|          | 60 | 80 |      |

$x$  - no of Ds

$y$  - no of S

$$2x + 4y \leq 80$$

$$x \geq 0$$

$$4x + 2y \leq 50$$

$$y \geq 0$$

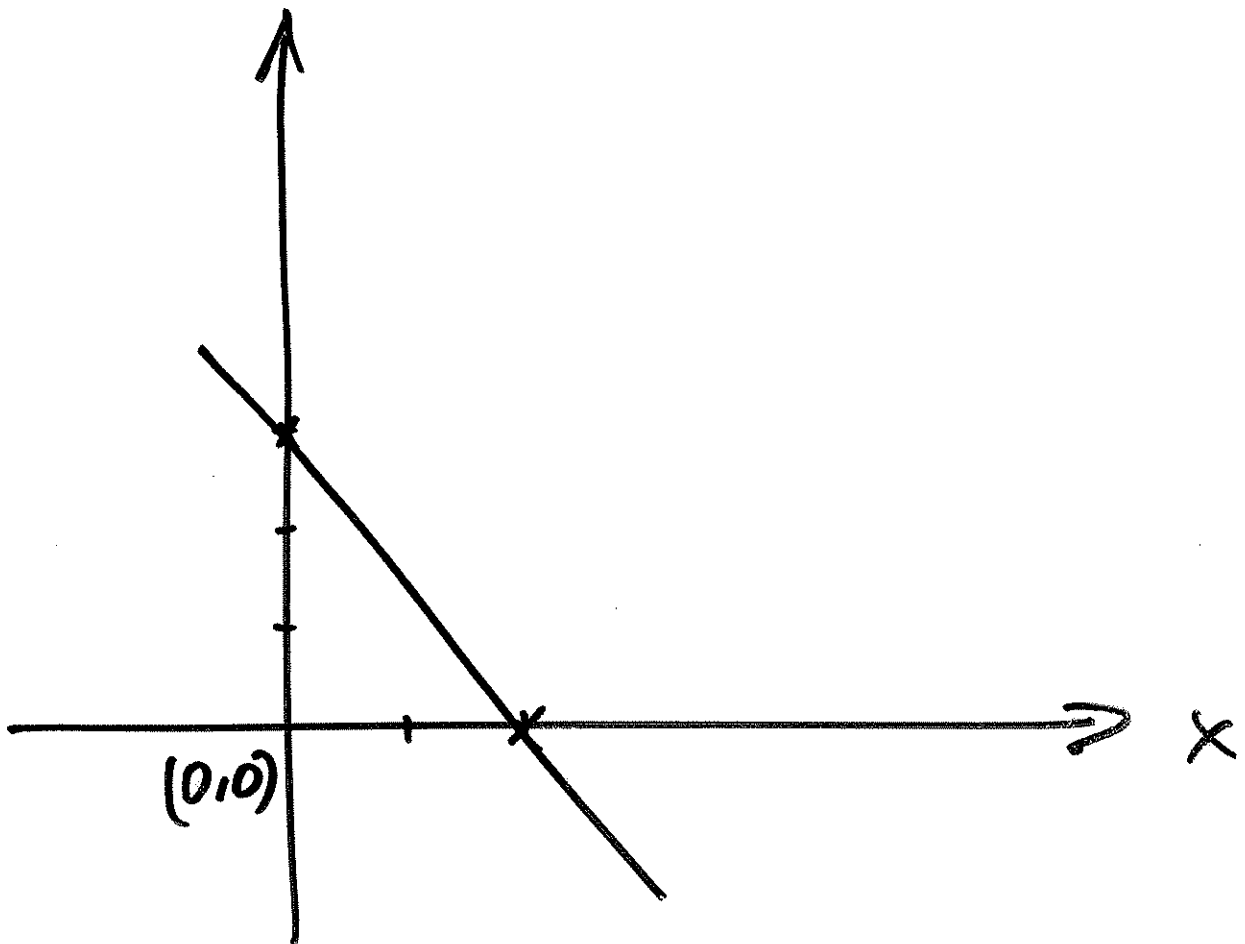
$$2x + 2y \leq 50$$

$$P = 60x + 80y$$

Graphing line

①

$$3x + 2y = 6$$



$$3x + 2y = 6$$

$$2y = -3x + 6$$

$$y = -\frac{3}{2}x + 3$$

$$x=0 \quad y=3$$

$$y=0 \quad x=2$$

②

$$3x + 4y \leq 18$$

$$2x + 3y \geq 12$$

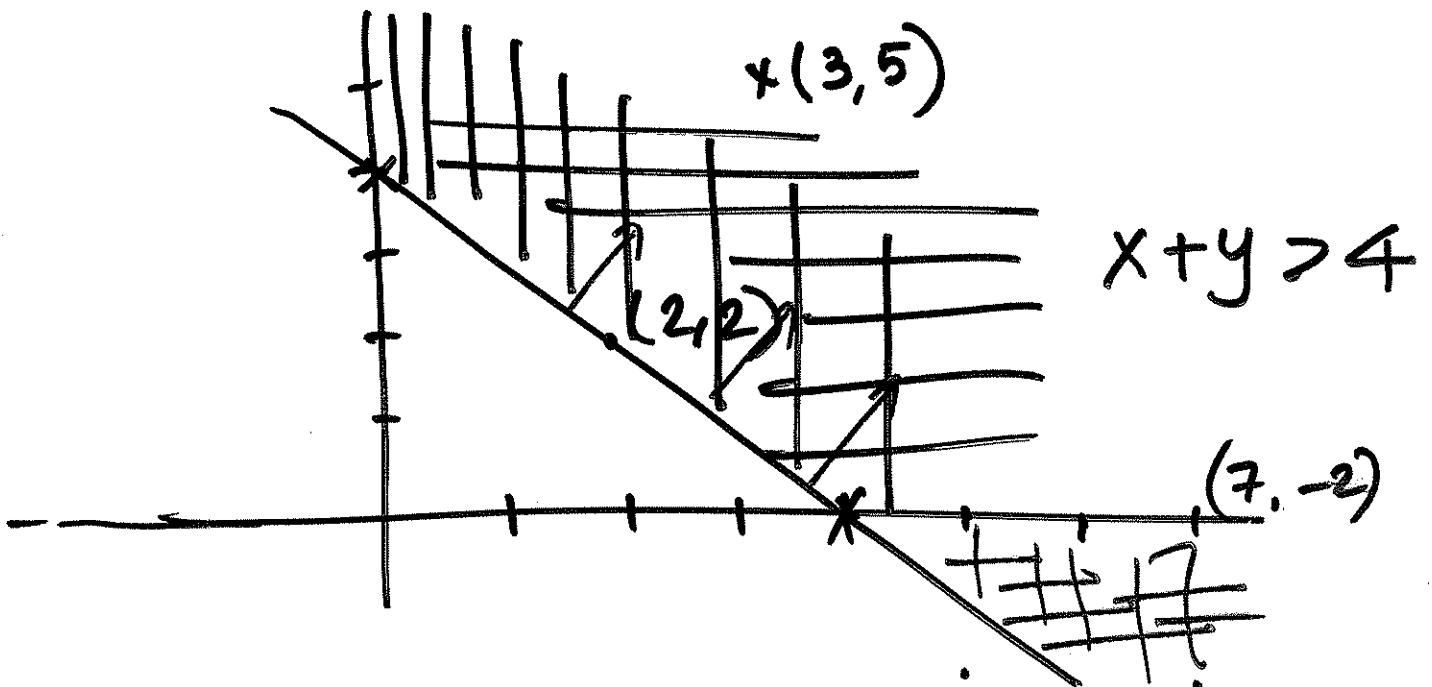
$$ax + by = c$$

$$ax + by < c \quad ax + by \leq c$$

$$ax + by \geq c \quad ax + by > c$$

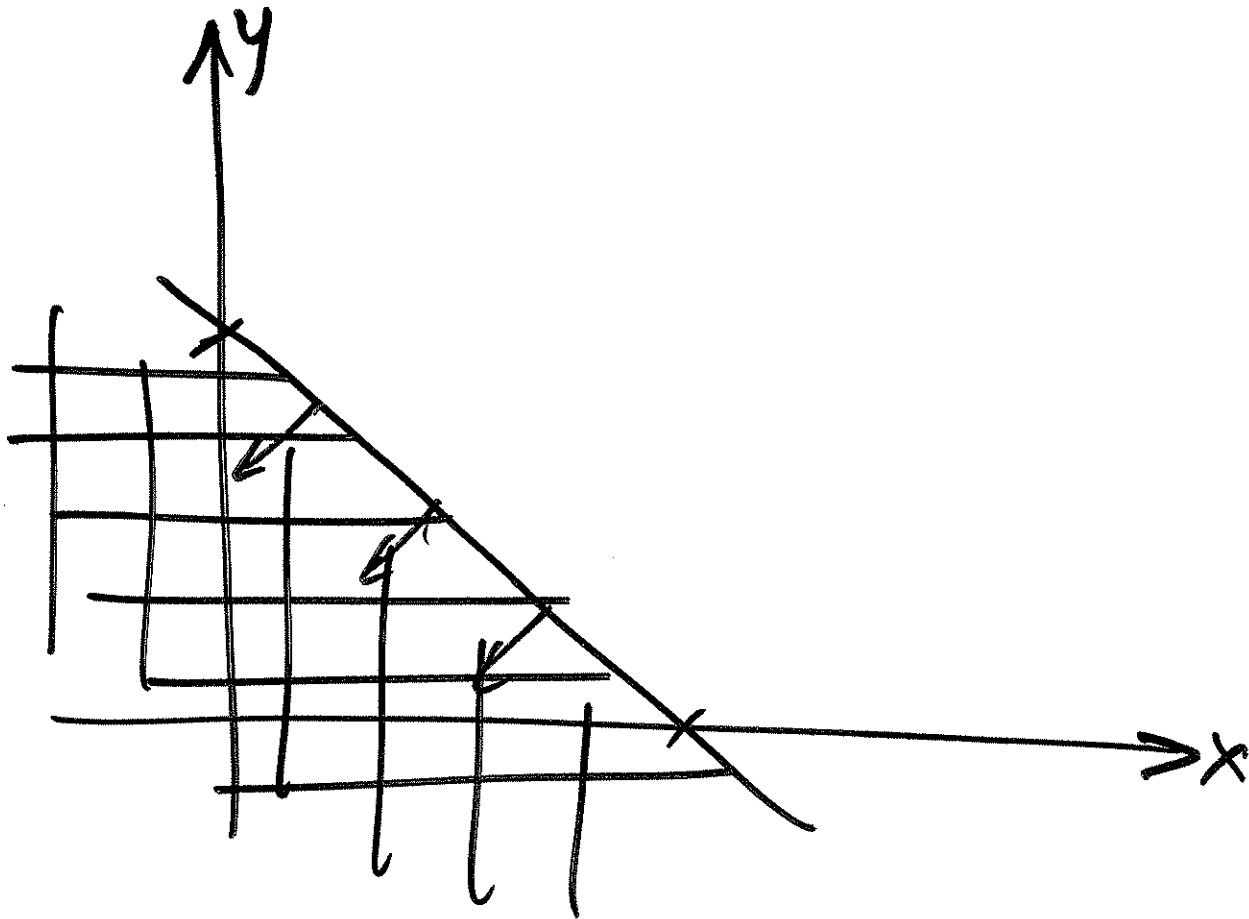
$$x + y = 4$$

$$x + y > 4$$



3

$$x + y < 4$$



$$4x + 4y \leq 32$$

$$x \geq 0$$

$$6x + 3y \leq 36$$

$$y \geq 0$$

$$4x + 4y = 32$$

$$4y = -4x + 32$$

$$y = -x + 8$$

$$6x + 3y = 36$$

$$3y = -6x + 36$$

$$y = -2x + 12$$

$$3x - 4y \leq 12$$

(4)

$$-4y \leq -3x + 12$$

$$-y \leq -\frac{3x}{4} + 3$$

$$y \geq +\frac{3x}{4} - 3$$

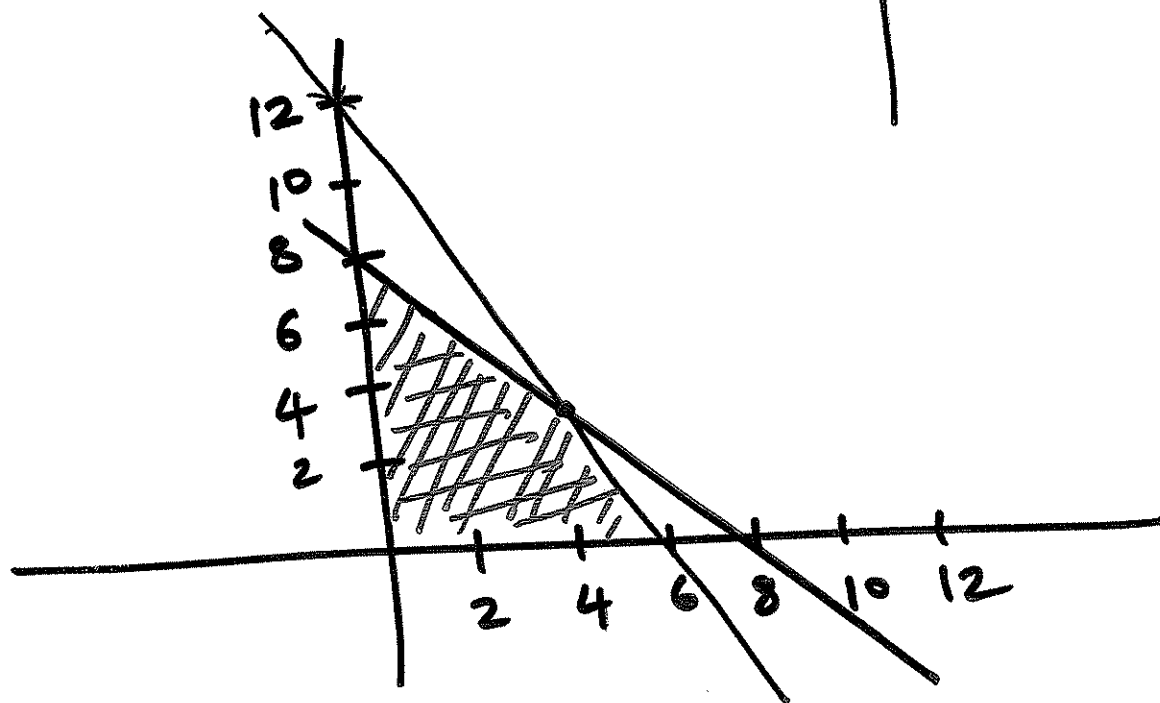
★ Remember to flip the sign when multiplying by -ve number

$$y \leq -x + 8$$

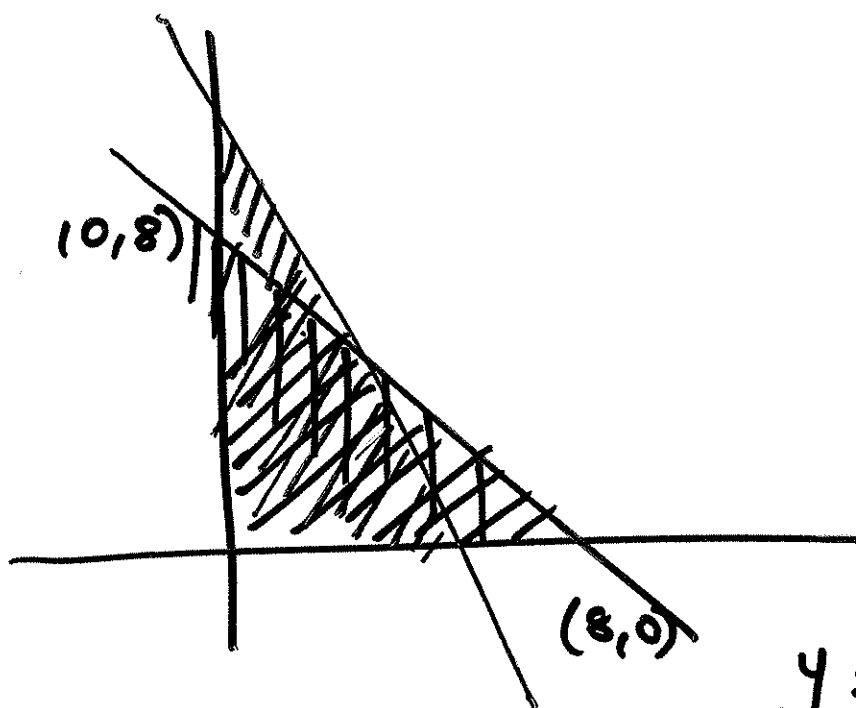
| y | x |
|---|---|
| 8 | 0 |
| 0 | 8 |

$$y \leq -2x + 12$$

| y  | x |
|----|---|
| 12 | 0 |
| 0  | 6 |



5



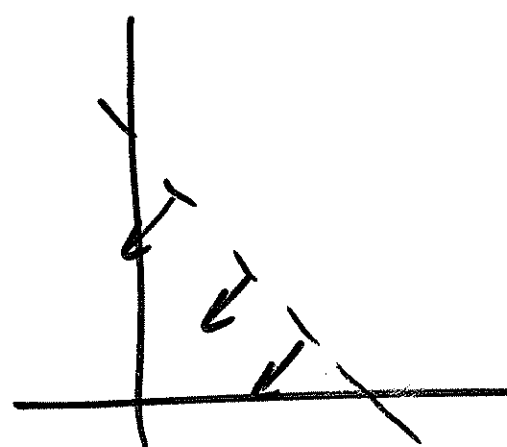
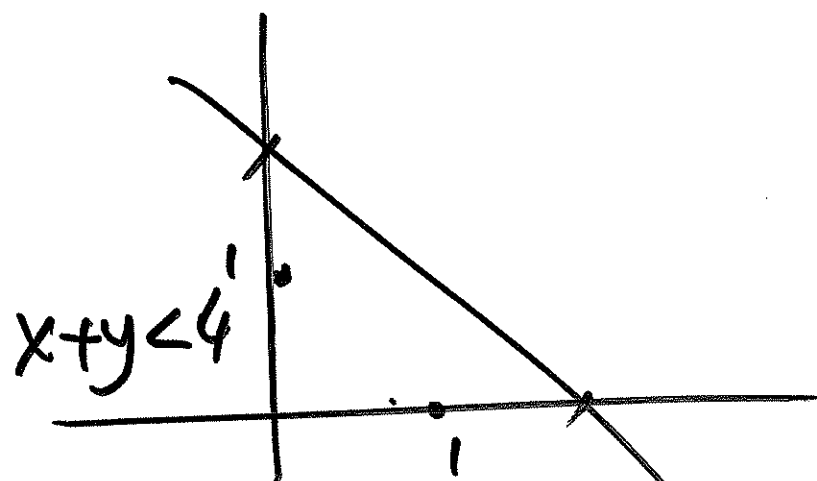
$$y \leq -x + 8$$

$$y \geq 0$$

$$x \geq 0$$

$$x + y \leq 4$$

$$x + y < 4$$



$$x + y = 4$$

$$x + y < 4$$

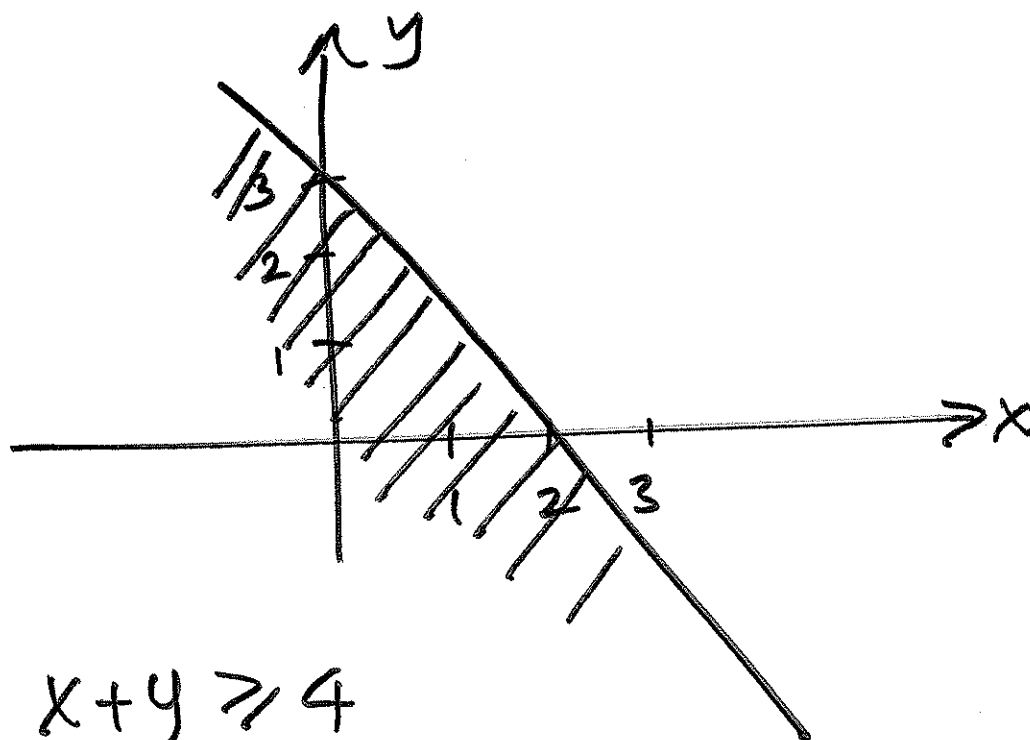
④  $3x + 2y \leq 6$

$$2y \leq -3x + 6$$

$$y \leq -\frac{3}{2}x + 3$$

$$y = -\frac{3}{2}x + 3$$

| y | x |
|---|---|
| 3 | 0 |
| 0 | 2 |



②⑥  $x + y \geq 4$   
 $-x + y \geq 2$

$$y \geq -x + 4$$

| y | x |
|---|---|
| 4 | 0 |
| 0 | 4 |

$$y \geq x + 2$$

| y | x  |
|---|----|
| 2 | 0  |
| 0 | -2 |

