

4.2 The number of Elements in a Set

Notation

$n(A)$ If A is a set with finite number of elements, we denote the number of elements in A by $n(A)$

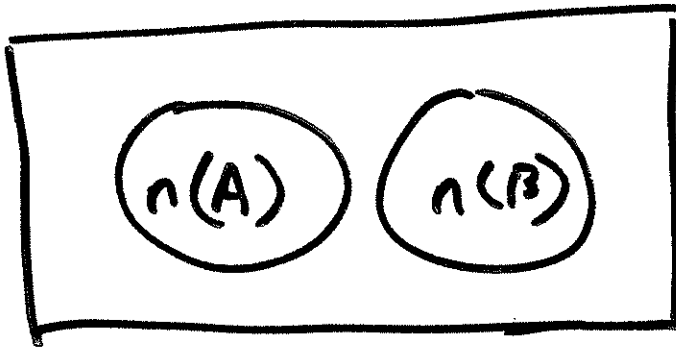
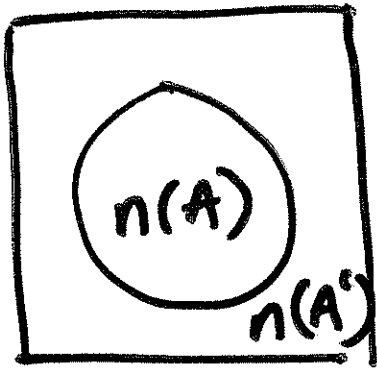
For Disjoint Set

$$n(A \cup B) = n(A) + n(B)$$

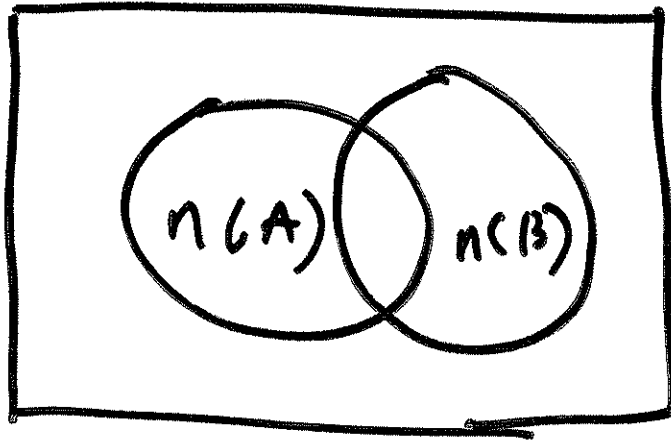
For not Disjoint Set

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

INCLUSION EXCLUSION PRINCIPLE



$$n(A \cup B) = n(A) + n(B)$$



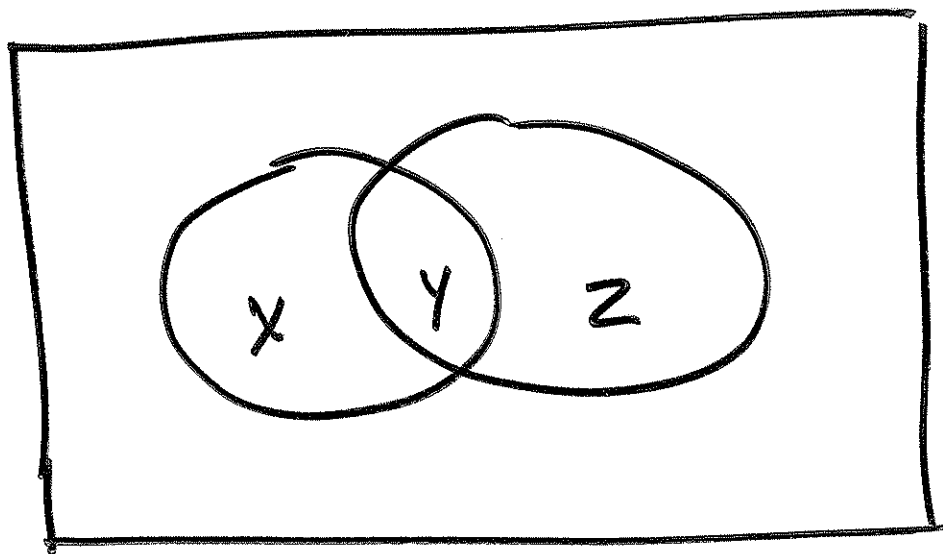
$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

Explain the formula w/ x, y, z .

3

$$n(A \cup B) = x + y + z$$

$$= (x + y) + (\cancel{y + z}) (y + z) - y$$



$$n(A \cup B) = x + y + z$$

$$= (x + y) + (y + z) - y$$

$$= n(A) + n(B) - n(A \cap B)$$

$$C \rightarrow 55$$

$$n(C) = 55 \quad (4)$$

$$\text{Eggs} \rightarrow 40$$

$$n(E) = 40$$

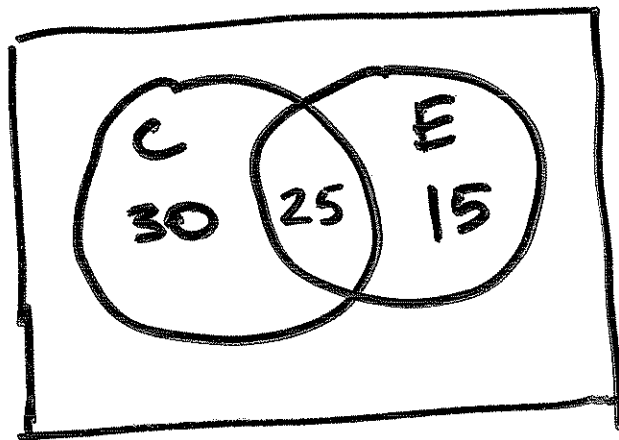
$$\text{Eggs \& } C \rightarrow 25$$

$$n(E \cap C) = 25$$

$$n(C)$$

$$n(E)$$

$$n(E \cap C) = 25$$



$$\begin{aligned} &X + Y + Z \\ &= 30 + 25 + 15 \\ &= 70 \end{aligned}$$

$$\text{only } C \rightarrow 55 - 25 = 30$$

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

$$n(E \cup C) = n(E) + n(C) - n(E \cap C)$$

$$= 35 + 30 - 25$$

$$= 20$$

3 sets

5

E - England

$$n(E) = 142$$

I - Italy

$$n(I) = 95$$

G - Germany

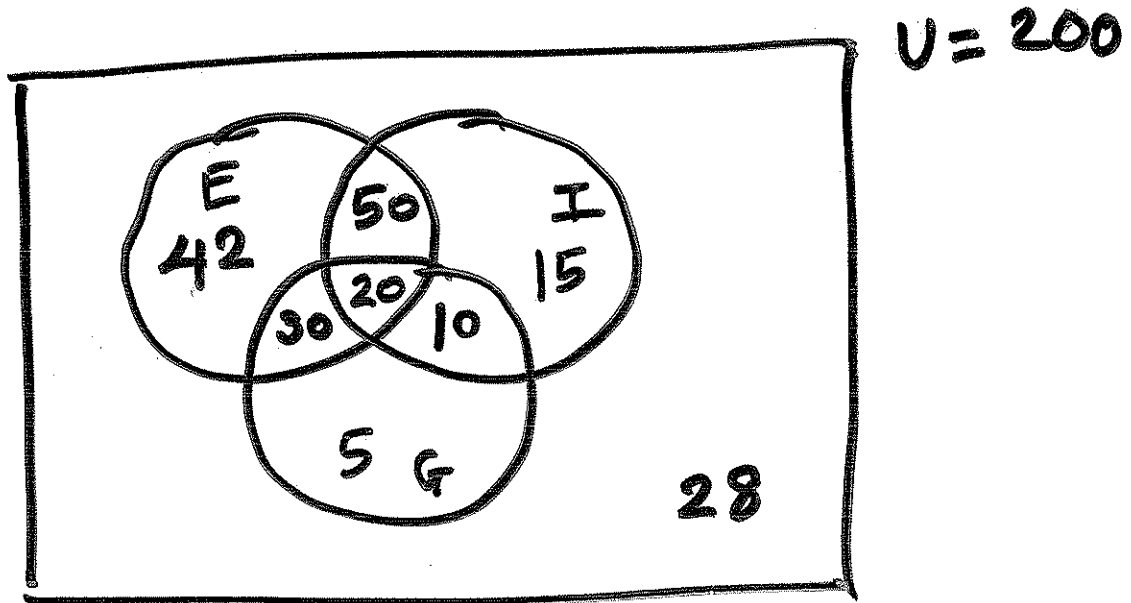
$$n(G) = 65$$

$$n(E \cap I \cap G) = 20$$

$$n(E \cap I) = 70$$

$$n(E \cap G) = 50$$

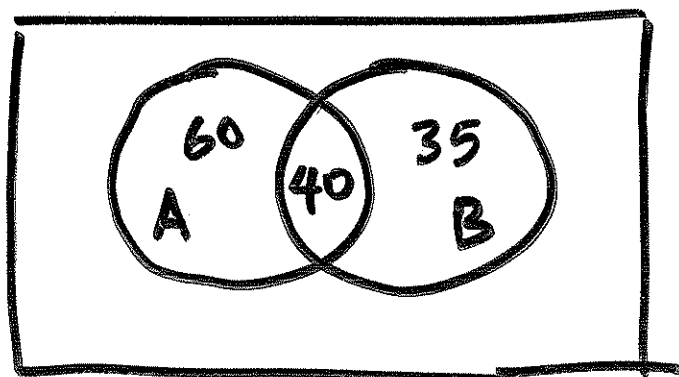
$$n(I \cap G) = 30$$



① If $n(A) = 100$ $n(A \cap B) = 40$
 $n(B) = 75$ $n(A \cup B)$

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

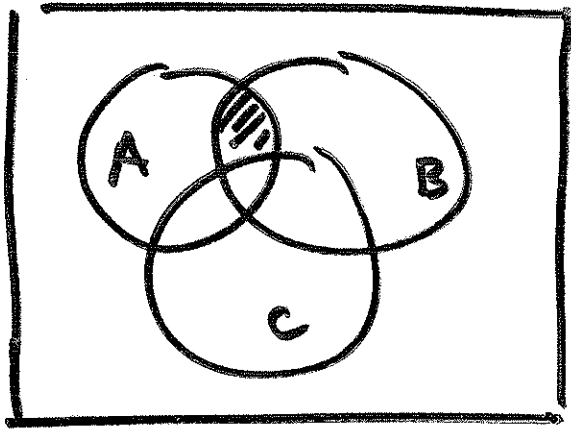
$$= 100 + 75 - 40 = 135$$



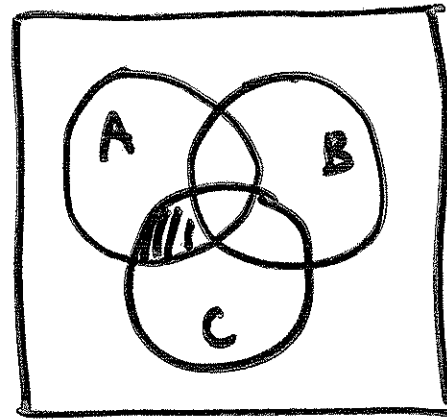
② $n(A) = 200$
 $n(A \cap B \cap C) = 40$
 $n(A \cap B \cap C^c) = 20$

$n(A \cap B^c \cap C) = 50$

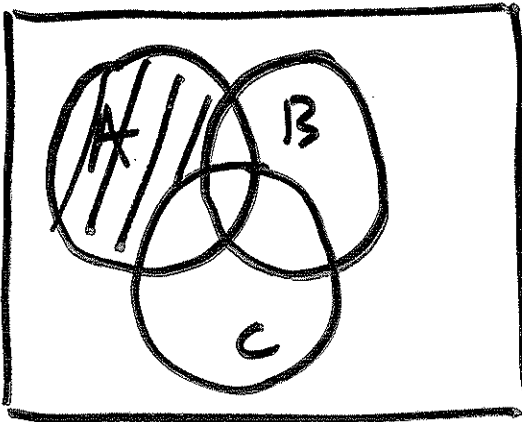
What is $n(A \cap B^c \cap C^c) = ?$



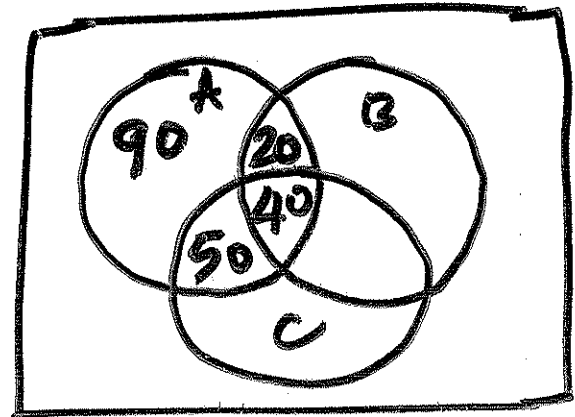
$$A \cap B \cap C^c = 20$$



$$A \cap B^c \cap C = 50$$



$$A \cap B^c \cap C^c = 90$$



$$n(A) = 200$$

$$\begin{aligned} n(A \cap B^c \cap C^c) &= n(A) - n(A \cap B \cap C) \\ &\quad - n(A \cap B \cap C^c) - n(A \cap B^c \cap C) \\ &= 200 - 20 - 20 - 50 \\ &= 90 \end{aligned}$$