

Chapter 4

Sets & Probability

Introduction to Sets

Key Concepts

- Definitions
- Set - notations
- meaning of Symbols
- Venn diagrams
- Laws

Set — A collection of items ^②

items — elements or member

Generally set is denote by

a capital letter

$$S = \{1, 2, 3, 4\}$$

$$1 \in S$$

← read as "is an element of"

$$C = \{1\}$$

C is subset of S

Subset

IF every element of a set A is also an element of another set B, we say that A is a subset of B and write

$$A \subseteq B$$

$$A \subset B$$

Example

$$A = \{1, 2, 4\} \quad B = \{1, 2, 3, 4\}$$

$$A \subseteq B$$

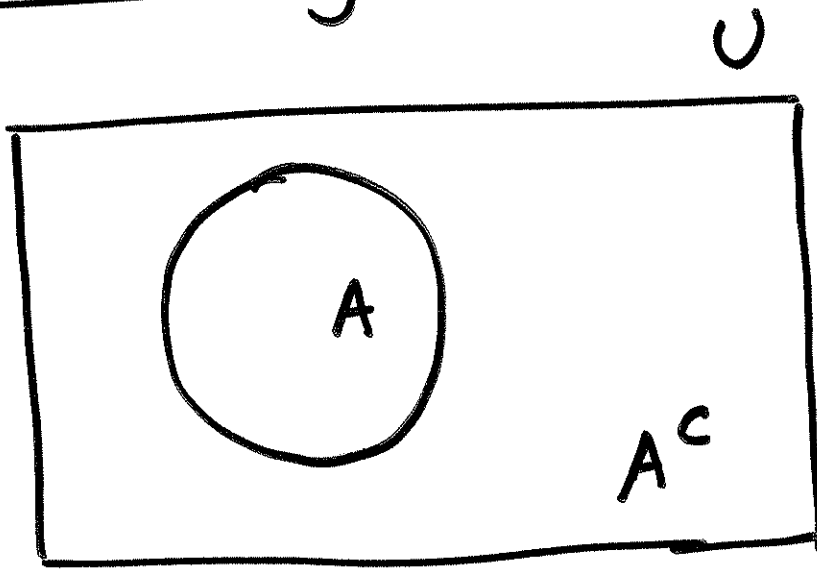
But $B \not\subseteq A$

Empty Set is denoted by

$$\emptyset \text{ or } \{\} \text{ Null}$$

Universal set has all elements

Venn diagram



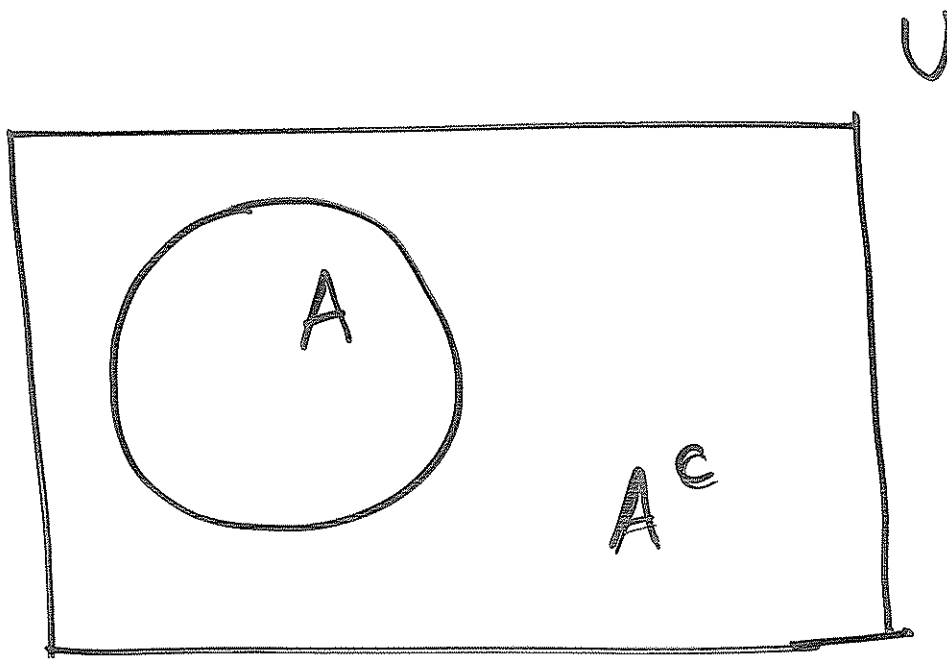
Complementary set

$$A^c = \{x \mid x \in U, x \notin A\}$$

In simple terms complementary set has all those elements which are not in the original set but they are in universal set

ex. $A = \{1, 2, 3\}$ $U = \{1, 2, 3, 4, 5, 6\}$

$$A^c = \{4, 5, 6\}$$



$$U = \{1, 2, 3, 4, 5, 6, 7, 8\}$$

$$A = \{1, 3, 5, 7\}$$

$$A^c = \{2, 4, 6, 8\}$$

$$A = \{ \text{All odd \# from 1 to 8} \}$$

$$A^c = \{ \text{All even \# from 1 to 8} \}$$

$A^c \rightarrow$ complement

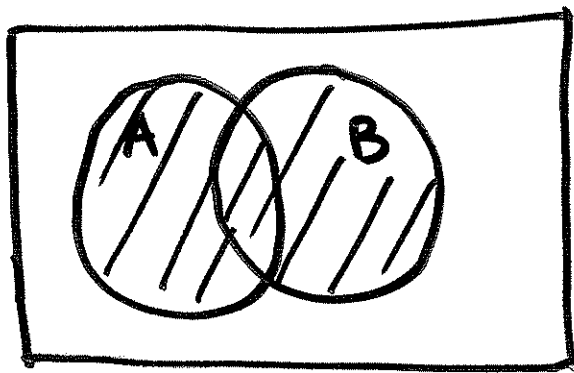
Complement Rules

$$U^c = \emptyset \quad \emptyset^c = U$$

$$(A^c)^c = A$$

Union

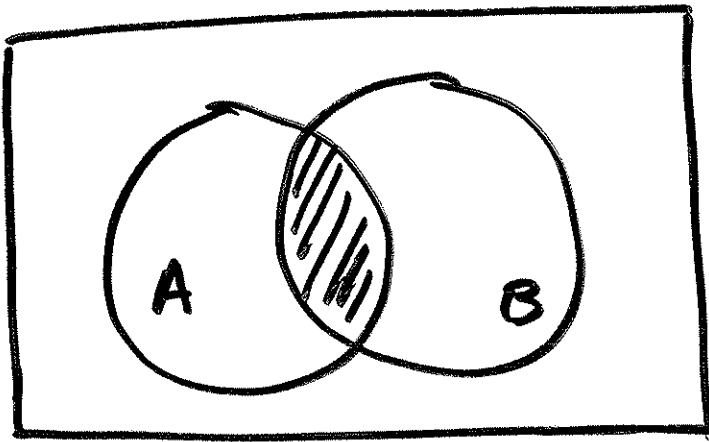
$A \cup B$ is the set of all elements that belong to A , or to B or to both



$A \cup B$

Intersection

6



$A \cap B$ Elements which belong
to A and B

$$U = \{1, 2, 3, 4\}$$

⑦

$$A = \{1, 2\}$$

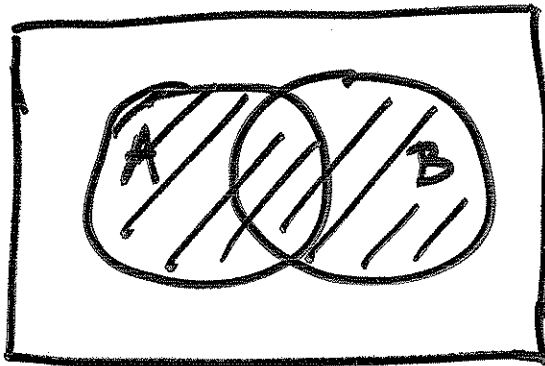
$$A^c = \{3, 4\}$$

$$(A^c)^c = \{1, 2\} = A$$

$A \cup B$
↑

union

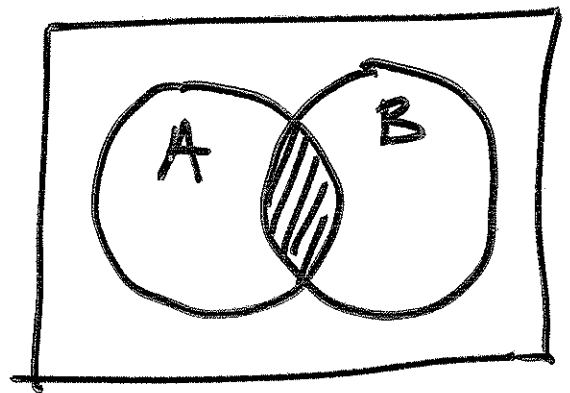
OR



$A \cap B$
↑

intersection

and

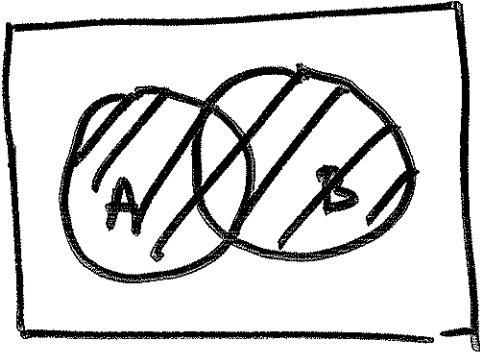


$$A = \{1, 2, 3, 4\} \quad B = \{1, 2, 4, 6, 7\}$$

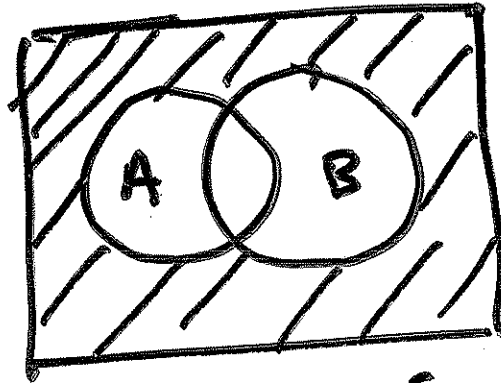
$$A \cup B = \{1, 2, 3, 4, 6, 7\}$$

$$A \cap B = \{1, 2, 4\}$$

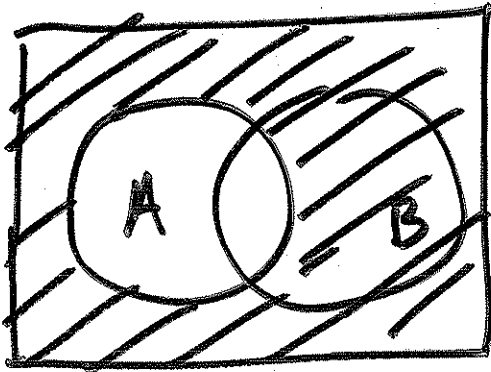
$$(A \cup B)^c = A^c \cap B^c$$



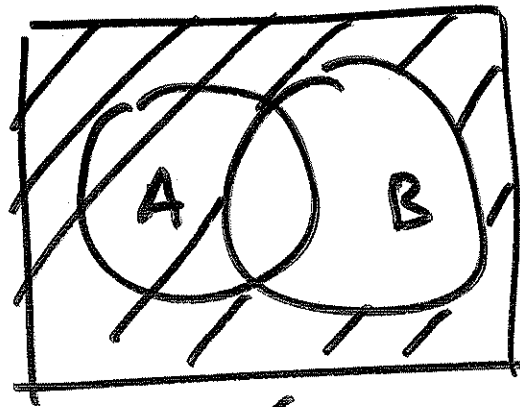
$A \cup B$



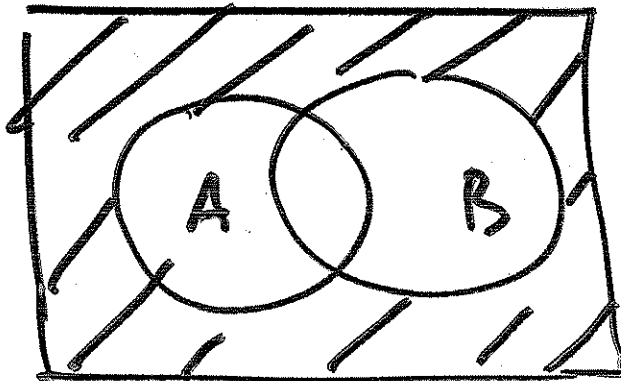
$(A \cup B)^c$



A^c



B^c

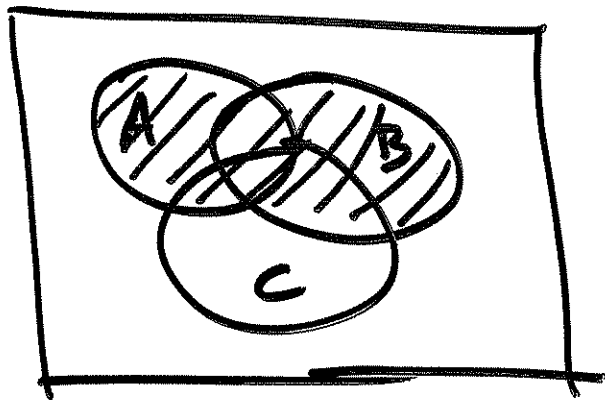


$A^c \cap B^c$

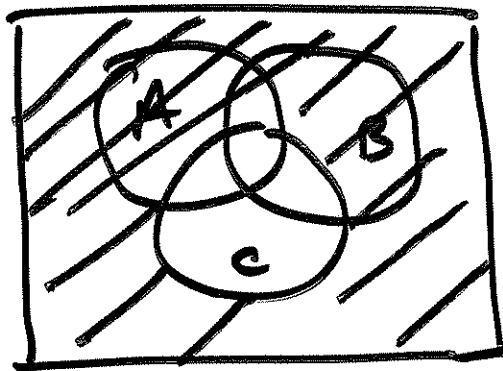
19

a

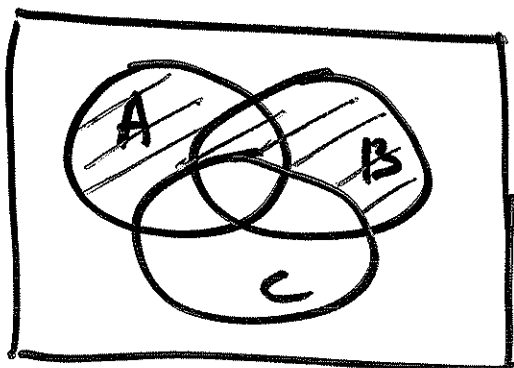
$$(A \cup B) \cap C^c$$



$A \cup B$



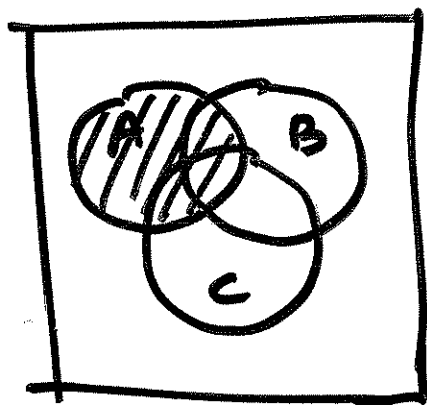
C^c



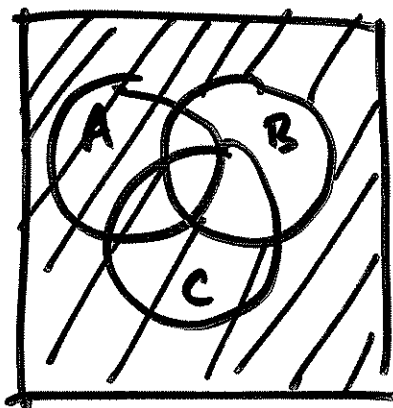
$(A \cup B) \cap C^c$

192¹⁵

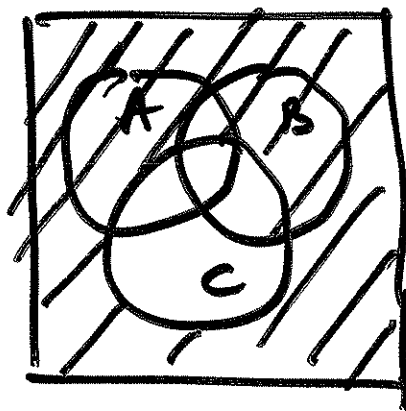
(b) $A \cap B^c \cap C^c$



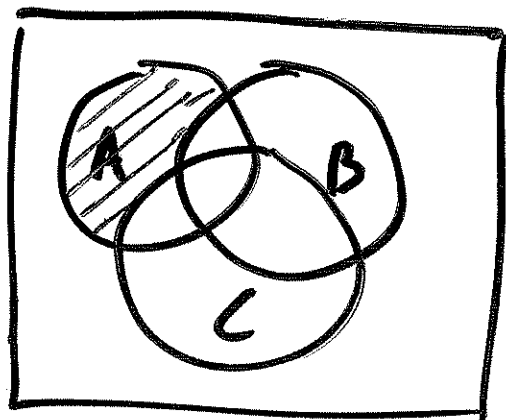
A



B^c



C^c



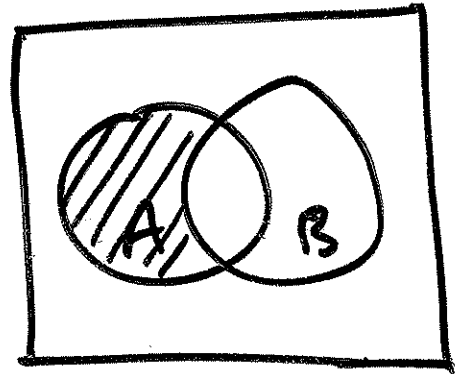
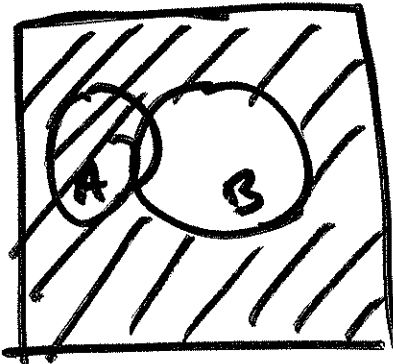
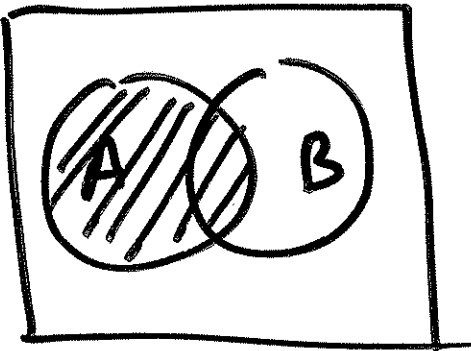
Disjoint Set

Page 189

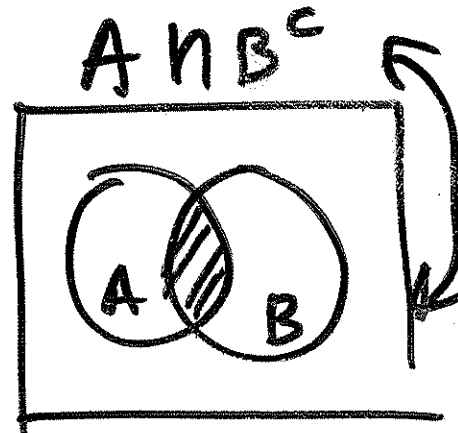
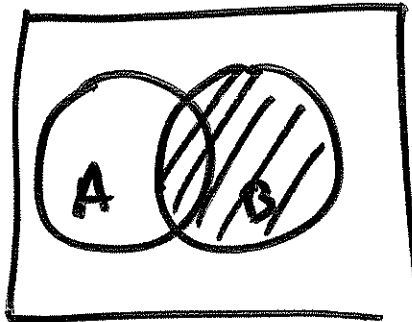
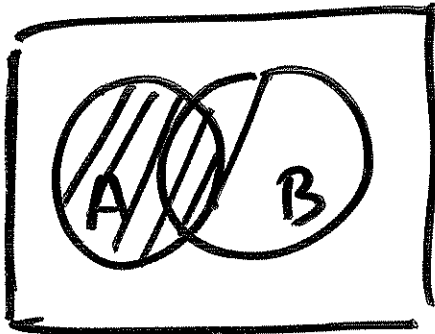
$$\text{if } A \cap B = \emptyset$$

(11) - 192

a)



b)



$$A \cap B$$

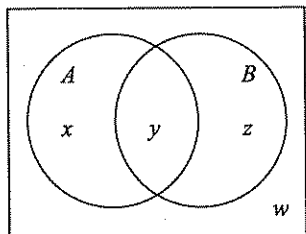
✧ Union Rule for Two Sets

Now consider the more general case shown in Figure 4.11. We assume that x is the number in the set A that are not in B , that is, $n(A \cap B^c)$. Next we have z , the number in the set B that are not in A , $n(A^c \cap B)$. Finally, y is the number in both A and B , $n(A \cap B)$ and w is the number of elements that are neither in A nor in B , $n(A^c \cap B^c)$. Then

$$\begin{aligned} n(A \cup B) &= x + y + z \\ &= (x + y) + (y + z) - y \\ &= n(A) + n(B) - n(A \cap B) \end{aligned}$$

Alternatively, we can see that the total $n(A) + n(B)$ counts the number in the intersection $n(A \cap B)$ twice. Thus to obtain the number in the union $n(A \cup B)$, we must subtract $n(A \cap B)$ from $n(A) + n(B)$.

Figure 4.11



The Number in the Union of Two Sets

For any finite sets A and B ,

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

EXAMPLE 1 An Application of Counting In a survey of 120 adults, 55 said they had an egg for breakfast that morning, 40 said they had juice for breakfast, and 70 said they had an egg or juice for breakfast. How many had an egg but no juice for breakfast? How many had neither an egg nor juice for breakfast?

Solution Let U be the universal set of adults surveyed, E the set that had an egg for breakfast, and J the set that had juice for breakfast. A Venn diagram is shown in Figure 4.12a. From the survey, we have that

$$n(E) = 55 \quad n(J) = 40, \quad n(E \cup J) = 70$$

Note that each of these is a sum. That is $n(E) = 55 = x + y$, $n(J) = 40 = y + z$ and $n(E \cup J) = 70 = x + y + z$. Since 120 people are in the universal set, $n(U) = 120 = x + y + z + w$.

The number that had an egg and juice for breakfast is given by $n(E \cap J)$ and is shown as the shaded region in Figure 4.12b. We apply the union rule:

$$\begin{aligned} n(E \cap J) &= n(E) + n(J) - n(E \cup J) \\ &= 55 + 40 - 70 \\ &= 25 \end{aligned}$$

We first place the number 25, just found, in the $E \cap J$ area in the Venn diagram in Figure 4.12b. Since the number of people who had eggs (with and without juice) is 55, then according to Figure 4.12b,

$$\begin{aligned} n(E) &= 55 = x + 25 \\ x &= 30 \end{aligned}$$

Figure 4.12a

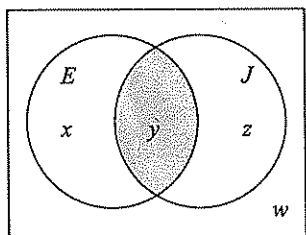
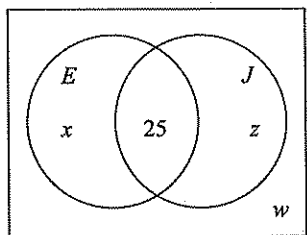


Figure 4.12b



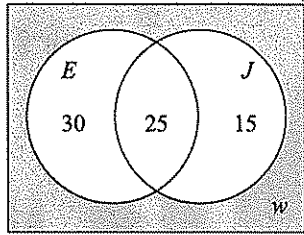


Figure 4.12c

Similarly, the number who had juice (with and without an egg) is 40. Using Figure 4.12b,

$$\begin{aligned} n(J) &= 40 = z + 25 \\ z &= 15 \end{aligned}$$

These two results are shown in Figure 4.12c. We wish to find $w = n((E \cup J)^c)$. This is shown as the shaded region in Figure 4.12c. The unshaded region is $E \cup J$. We then have that

$$\begin{aligned} n(E \cup J) + n((E \cup J)^c) &= n(U) \\ n((E \cup J)^c) &= n(U) - n(E \cup J) \\ w &= 120 - 70 \\ w &= 50 \end{aligned}$$

And so there were 50 people in the surveyed group that had neither an egg nor juice for breakfast. ♦

✧ Counting With Three Sets

Many counting problems with sets have two sets in the universal set. We will also study applications with three sets in the universal set. The union rule for three sets is studied in the extensions for this section. In the example below, deductive reasoning is used to solve for the number of elements in each region of the Venn diagram. In cases where this will not solve the problem, systems of linear equations can be used to solve the Venn diagram. This is studied in the Chapter Project found in the Review section.

EXAMPLE 2 European Travels In a survey of 200 people that had just returned from a trip to Europe, the following information was gathered.

- 142 visited England
- 95 visited Italy
- 65 visited Germany
- 70 visited both England and Italy
- 50 visited both England and Germany
- 30 visited both Italy and Germany
- 20 visited all three of these countries

- a. How many went to England but not Italy or Germany?
- b. How many went to exactly one of these three countries?
- c. How many went to none of these three countries?

Solution Let U be the set of 200 people that were surveyed and let

$E = \{x | x \text{ visited England}\}$

$I = \{x | x \text{ visited Italy}\}$

$G = \{x | x \text{ visited Germany}\}$

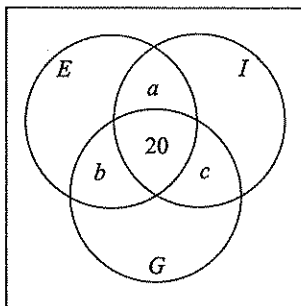


Figure 4.13a

We first note that the last piece of information from the survey indicates that

$$n(E \cap I \cap G) = 20$$

Place this in the Venn diagram shown in Figure 4.13a. Recall that 70 visited both England and Italy, that is, $n(E \cap I) = 70$. If a is the number that visited England and Italy but not Germany, then, according to Figure 4.13a, $20 + a = n(E \cap I) = 70$. Thus $a = 50$. In the same way, if b is the number that visited England and Germany but not Italy, then $20 + b = n(E \cap G) = 50$. Thus $b = 30$. Also if c is the number that visited Italy and Germany but not England, then $20 + c = n(G \cap I) = 30$. Thus $c = 10$. All of this information is then shown in Figure 4.13b.

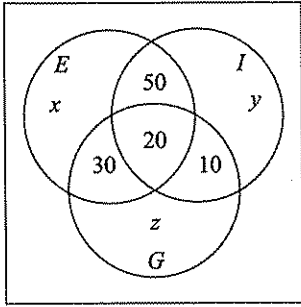


Figure 4.13b

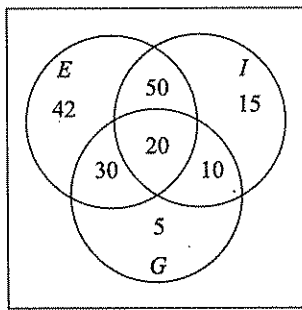


Figure 4.13c

- Let x denote the number that visited England but not Italy or Germany. Then, according to Figure 4.13b, $20 + 30 + 50 + x = n(E) = 142$. Thus $x = 42$, that is, the number that visited England but not Italy or Germany is 42.
- Since $n(I) = 95$, the number that visited Italy but not England or Germany is given from Figure 4.13b by $95 - (50 + 20 + 10) = 15$. Since $n(G) = 65$, the number that visited Germany but not England or Italy is, according to Figure 4.13b, given by $65 - (30 + 20 + 10) = 5$. Thus, according to Figure 4.13c, the number who visited just one of the three countries is

$$42 + 15 + 5 = 62$$

- There are 200 people in the U and so according to Figure 4.13c, the number that visited none of these three countries is given by

$$200 - (42 + 15 + 5 + 50 + 30 + 10 + 20) = 200 - 172 = 28$$

EXAMPLE 3 Pizzas At the end of the day the manager of Blue Baker wanted to know how many pizzas were sold. The only information he had is listed below. Use the information to determine how many pizzas were sold.

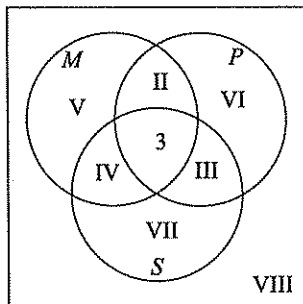


Figure 4.14a

- 3 pizzas had mushrooms, pepperoni, and sausage
- 7 pizzas had pepperoni and sausage
- 6 pizzas had mushrooms and sausage but not pepperoni
- 15 pizzas had two or more of these toppings
- 11 pizzas had mushrooms
- 8 pizzas had only pepperoni
- 24 pizzas had sausage or pepperoni
- 17 pizzas did not have sausage

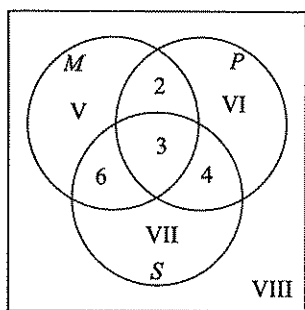


Figure 4.14b

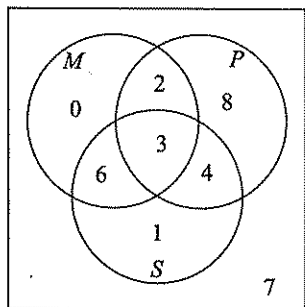


Figure 4.14c

Solution Begin by drawing a Venn diagram with a circle for pizzas that had mushrooms, a circle for pizzas that had pepperoni, and, pizzas that had sausage. In the center place a 3 since three pizzas had all these toppings. See Figure 4.14a.

Since 7 pizzas have pepperoni and sausage, $7 = 3 + \text{III}$ or $\text{III} = 4$. If 6 pizzas had mushrooms and sausage but not pepperoni, then $\text{IV} = 6$. The region for two or more of these toppings is $3 + \text{II} + \text{III} + \text{IV} = 15$. Using $\text{III} = 4$ and $\text{IV} = 6$, that gives $3 + \text{II} + 4 + 6 = 15$ or $\text{II} = 2$. This information is shown in Figure 4.14b.

Given that 11 pizzas had mushrooms, $\text{V} + 2 + 3 + 6 = 11$ and therefore $\text{V} = 0$. Since 8 pizzas had only pepperoni, $\text{VI} = 8$. With a total of 24 pizzas in the sausage or pepperoni region and knowing that $\text{VI} = 8$, we have $2 + 8 + 6 + 3 + 4 + \text{VII} = 24$ or $\text{VII} = 1$. Finally, if 17 pizzas did not have sausage then $17 = \text{V} + 2 + \text{VI} + \text{VIII} = 0 + 2 + 8 + \text{VIII}$. This gives $\text{VIII} = 7$ and our complete diagram is shown in Figure 4.14c.

To find the total number of pizzas sold, the 8 numbers in the completed Venn diagram are added:

$$0 + 2 + 8 + 6 + 3 + 4 + 1 + 7 = 31$$



Self-Help Exercises 4.2

- Given that $n(A \cup B) = 100$, $n(A \cap B^c) = 50$, and $n(A \cap B) = 20$, find $n(A^c \cap B)$.
- The registrar reported that among 2000 students, 700 did not register for a math or English course, while 400 registered for both of these two courses. How many registered for exactly one of these courses?
- One hundred shoppers are interviewed about the contents of their bags and the following results are found:
 - 40 bought apple juice

- 19 bought cookies
- 13 bought broccoli
- 1 bought broccoli, apple juice, and cookies
- 11 bought cookies and apple juice
- 2 bought cookies and broccoli but not apple juice
- 24 bought only apple juice

Organize this information in a Venn diagram and find how many shoppers bought none of these items.

4.2 Exercises

- If $n(A) = 100$, $n(B) = 75$, and $n(A \cap B) = 40$, what is $n(A \cup B)$?
- If $n(A) = 200$, $n(B) = 100$, and $n(A \cup B) = 250$, what is $n(A \cap B)$?
- If $n(A) = 100$, $n(A \cap B) = 20$, and $n(A \cup B) = 150$, what is $n(B)$?
- If $n(B) = 100$, $n(A \cup B) = 175$, and $n(A \cap B) = 40$, what is $n(A)$?